KINEMATIC IDENTIFICATION OF PARALLEL MECHANISMS BY A DIVIDE AND CONQUER STRATEGY

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Abstract: This paper presents a Divide and Conquer strategy to estimate the kinematic parameters of parallel symmetrical mechanisms. The Divide and Conquer kinematic identification is designed and performed independently for each leg of the parallel mechanism. The estimation of the kinematic parameters is performed using the inverse calibration method. The identification poses are selected optimizing the observability of the kinematic parameters from the Jacobian identification matrix. With respect to traditional identification methods the main advantages of the proposed Divide and Conquer kinematic identification strategy are: (i) reduction of the kinematic identification computational costs, (ii) improvement of the numerical efficiency of the kinematic identification algorithm and, (iii) improvement of the kinematic identification results. The contributions of the paper are: (i) The formalization of the inverse calibration method as the Divide and Conquer strategy for the kinematic identification of parallel symmetrical mechanisms and, (ii) a new kinematic identification protocol based on the Divide and Conquer strategy. As an application of the proposed kinematic identification protocol the identification of a planar 5R symmetrical mechanism is (virtually) developed. The performance of the calibrated mechanism is evaluated by updating the kinematic models with the estimated parameters and developing kinematic simulations.

1 INTRODUCTION

In mechanisms and manipulators the accuracy of the end-effector critically depends on the knowledge of the kinematic model governing the control model (Zhuang et al., 1998). Therefore, to improve the accuracy of a mechanism its kinematic parameters have to be precisely estimated (Renaud et al., 2006). The process of estimating the kinematic parameters and updating the kinematic model is formally known as kinematic identification or kinematic calibration (Merlet, 2006).

Kinematic identification is an instance of the robot calibration problem. The estimation of rigid-body inertial parameters and the estimation of sensor gain and offset are instances of calibration problems at the same hierarchical level of the kinematic calibration problem (Hollerbach et al., 2008).

This paper is devoted to the kinematic identification of parallel symmetrical mechanisms. Parallel mechanisms are instances of closed-loop mechanisms typically formed by a moving platform connected to a fixed base by several legs. Each leg is a kinematic chain formed by a pattern of links, actuated and passive joints relating the moving platform with the fixed base. If the pattern of joints and links is the same for each leg and each leg is controlled by one actuator then the parallel mechanism is called symmetrical (Tsai, 1999). Most of the industrial parallel mechanisms can be classified as parallel symmetrical mechanisms.

For parallel mechanisms the kinematic identification is usually performed minimizing an error between the measured joint variables and their corresponding values calculated from the measured end-effector pose through the inverse kinematic model (Zhuang et al., 1998; Renaud et al., 2006). This method is preferred for the identification of parallel mechanisms because:

1. Inverse kinematics of parallel mechanisms is usu-

ally derived analytically avoiding the numerical problems associated with any forward kinematics solution (Zhuang et al., 1998; Renaud et al., 2006).

2. The inverse calibration method is considered to be the most numerically efficient among the identification algorithms for parallel mechanisms (Renaud et al., 2006; Besnard and Khalil, 2001) and,
3. With respect to forward kinematic identification no scaling is necessary to balance the contribution of position and orientation measurements (Zhuang et al., 1998).

In the case of parallel symmetrical mechanisms the inverse kinematic modeling can be formulated using independent loop-closure equations. Each loop-closure equation relates the end-effector pose, the geometry of a leg and, a fixed reference frame. In consequence, an independent kinematic constraint equation is formulated for each leg forming the mechanism. For the case of parallel symmetrical mechanisms the set of constraint equations is equal to the number of legs and to the number of degrees of freedom of the mechanisms. Each kinematic constraint equation can be used for the independent identification of the parameters of the leg correspondent to the equation.

The independent identification of the kinematic parameters of each leg in parallel mechanisms allows to improve:

1. The numerical efficiency of the identification algorithm (Zhuang et al., 1998) and,
2. The kinematic calibration performance by the design of independent experiments optimized for the identification of each leg.

The independent identification of leg parameters in parallel mechanisms was sketched in (Zhuang et al., 1998) and developed for the specific case of Gough platforms in (Daney et al., 2002; Daney et al., 2005). However, the idea of the independence in the kinematic identification of each leg in a parallel mechanism is not completely formalized.

This article presents a contribution to the improvement of the pose accuracy in parallel symmetrical mechanisms by a kinematic calibration protocol based on inverse kinematic modeling and a divide and conquer strategy. The proposed divide and conquer strategy takes advantage of the independent kinematic identification of each leg in a parallel mechanism not only from a numerical stand point but also from the selection of the optimal measurement set of poses that improves the kinematic identification of the parameters of the leg itself.

The layout for the rest of the document is as follows: section 2 develops a literature review on the inverse calibration of parallel mechanisms method, section 3 presents the divide and conquer identification of parallel mechanisms strategy, section 4 develops the kinematic identification of parallel mechanisms protocol, section 5 presents the results of the kinematic identification of a planar 5R symmetrical mechanism by computer simulations based on the identification protocol, finally, in section 6 the conclusions are developed.

2 LITERATURE REVIEW

The modeling of mechanical systems includes the design, analysis and control of mechanical devices. An accurate identification of the model parameters is required in the case of control tasks (Hollerbach et al., 2008). Instances of models of mechanical systems includes kinematic, dynamic, sensor, actuators and flexibility models. For parallel mechanisms, updating the kinematic models with accurately estimated parameters is essential to achieve precise motion at high-speed rates. This is the case when parallel mechanisms are used in machining applications (Renaud et al., 2006).

The the inverse calibration method is accepted as the natural (Renaud et al., 2006; Zhuang et al., 1998) and most numerically efficient among the identification algorithms for parallel mechanisms (Renaud et al., 2006; Besnard and Khalil, 2001). The inverse calibration method is based on inverse kinematic modeling and a external metrological system. The calibration is developed minimizing an error residual between the measured joint variables and its estimated values from the end-effector pose though the inverse kinematic model. The derivation of the inverse kinematic model of parallel mechanisms is usually straightforward obtained (Merlet, 2006). Kinematic identification of parallel mechanisms based on inverse kinematics and the use of external metrology is reported in: Systematic approaches (Merlet, 2006; Renaud et al., 2006; Iuraşcu and Park, 2003), calibration of hexapod mechanisms (Zhuang et al., 1998; Koseki et al., 1998; Zhuang et al., 1995; Huang et al., 2005), calibration of parallel mechanisms based machine-tools (Chanal et al., 2007; Wang and Fan, 2004), calibration of an orthoglide parallel mechanism (Pashkevich et al., 2009), calibration of redundant parallel mechanisms (il Jeong et al., 2004), calibration of a microparallel mechanism (Kang et al., 2008), calibration of parallel mechanisms based on inverse kinematics singularities (type 2 singularities) (Last and Hesselbach, 2006), calibration of parallel mechanisms with Denavit and Hartenberg kinematic modeling (Ha, 2008), identifiability of kinematic parameters (Daney et al., 2006b), and vision based identification (Daney et al., 2006a).

For the divide and conquer kinematic calibration strategy we adopt the inverse calibration method. The method takes advantage of an intrinsic characteristic of parallel mechanisms: the straightforward calculation of the inverse kinematics. However, not all the intrinsic characteristics of parallel mechanisms are exploited. Specifically, (Zhuang et al., 1998; Ryu and Rauf, 2001) reported that for parallel mechanism, methods based on inverse kinematics allow to identify error parameters of each leg of the mechanism independently. The independent parameter identification of each leg is reported to improve the numerical efficiency of the kinematic identification algorithm, (Zhuang et al., 1998). However, it is not reported a general kinematic identification strategy based on the independent identification of the legs and its advantages with respect to traditional identification methods.

This article presents a contribution to the kinematic calibration of parallel mechanisms developing a kinematic identification protocol based on the inverse calibration method and the independent identification of the parameters of each leg (Divide and Conquer strategy).

With respect to traditional identification methods, our Divide and Conquer strategy has the following advantages:

1. The identification poses can be optimized to the identification of reduced sets of parameters (the sets corresponding to each leg),
2. The independent identification of the parameters of each leg improves the numerical efficiency of the the identification algorithms and,
3. By 1 and 2 the identified set of parameters is closer to the real (unknown) set of parameters than sets identified by other traditional calibration methods.

The divide and Conquer strategy for the independent kinematic identification of the parameters of each leg in a parallel symmetrical mechanism is presented in section 3.

3 DIVIDE AND CONQUER IDENTIFICATION STRATEGY

Parallel symmetrical mechanisms satisfy (Tsai, 1999):

1. The number of legs is equal to the number of degrees of freedom of the end-effector.
2. All the legs have an identical structure. This is, each leg has the same number of active and passive joints and the joints are arranged in an identical pattern.

In a practical way, the definition of parallel symmetrical mechanism covers most of the industrial parallel structures. For parallel symmetrical mechanisms the kinematic identification by inverse kinematics and a divide and conquer strategy is stated as:

Given

1. A set of nominal kinematic parameters (φ) of the mechanism in terms of the parameters of the individual legs (φ_κ):

$$\begin{aligned}\varphi_\kappa = \left[\varphi_{\kappa,1} \quad \dots \varphi_{\kappa,n_{\varphi limb}}\right]^T \\ \kappa = 1,2,\dots,n_{limbs}\end{aligned} \tag{1}$$

2. An inverse kinematic function g_κ relating the κth active joint variables $(\mathbf{q}_\kappa)$ with the end-effector pose $(\mathbf{r})$. For the jth pose of the mechanism the inverse function of the κth leg is defined to be:

$$\begin{aligned} g_\kappa^j : \varphi_\kappa, \mathbf{r}^j \to \mathbf{q}_\kappa^j \\ \kappa = 1,2,\dots,n_{limbs} \\ j = 1,2,\dots,N \end{aligned} \tag{2}$$

3. A set of N measured end-effector configurations in terms of the independent identification of the legs $(\hat{\mathbf{R}}_\kappa)$.

$$\begin{aligned}\hat{\mathbf{R}}_\kappa = \left[\hat{\mathbf{r}}_\kappa^1 \quad \cdots \quad \hat{\mathbf{r}}_\kappa^N\right]^T, \\ \kappa = 1,2,\dots n_{limbs}\end{aligned} \tag{3}$$

4. A set of measured input variables $\hat{\mathbf{Q}}_\kappa$ corresponding to $\hat{\mathbf{R}}_\kappa$ in terms of the active joint variables of the κth leg:

$$\begin{aligned}\hat{\mathbf{Q}}_\kappa = \left[\hat{\mathbf{q}}_\kappa^1 \quad \cdots \quad \hat{\mathbf{q}}_\kappa^N\right]^T \\ \kappa = 1,2,\dots n_{limbs}\end{aligned} \tag{4}$$

Goal

To find the set of unknown (real) kinematic parameters $(\hat{\varphi}_\kappa)$ that minimizes an error between the measured joint variables $(\hat{\mathbf{Q}}_\kappa)$ and their corresponding values $(\bar{\mathbf{Q}}_\kappa)$, estimated from the measured end-effector pose by the inverse kinematic model g_κ. The problem can be formally stated as the following nonlinear minimization problem:

$$\begin{aligned}\hat{\varphi}_\kappa : \sum_{N}^{j=1} \left\|\hat{\mathbf{Q}}_\kappa - \bar{\mathbf{Q}}_\kappa\left(\hat{\mathbf{R}}_\kappa, \hat{\varphi}_\kappa\right)\right\|^2 \text{is minimal.} \\ \text{subject to} : \mathbf{R}_\kappa \subset \mathbf{W_R} \\ \mathbf{W_R} \text{ is the usable end} - \text{effector workspace} \\ \kappa = 1,2,\dots,n_{limbs}\end{aligned} \tag{5}$$

The optimization problem is constrained by the workspace of the mechanism. The usable workspace is defined as the workspace without singularity by (Liu et al., 2006a).

A kinematic identification of parallel symmetrical mechanisms protocol based on the Divide and Conquer identification strategy is developed in section 4.

4 KINEMATIC IDENTIFICATION PROTOCOL

Based on the Divide and Conquer strategy for the kinematic identification of parallel symmetrical mechanisms (section 3) the following kinematic identification protocol (Fig. 1) is proposed.

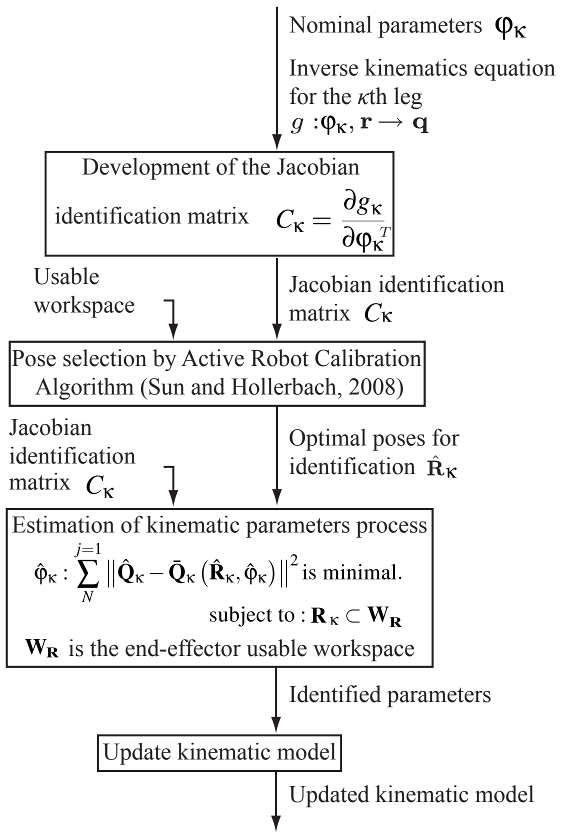

Figure 1: Kinematic identification of parallel symmetrical mechanisms protocol.

1. Given the nominal parameters of the κth leg (φ_κ, Eq. 1) and the correspondent inverse kinematic function (g_κ, Eq. 2) to develop the κth Jacobian identification matrix as:

$$C_\kappa = \frac{\partial g_\kappa}{\partial \varphi_\kappa^T} \tag{6}$$

2. Given the Jacobian identification matrix developed in 1 and the usable workspace ($\mathbf{W_R}$) of the mechanism to select an optimal set of postures ($\mathbf{R}_\kappa$) for the kinematic identification of the κth leg. Each set of postures is selected searching the improvement of the observability of the set of parameters φ_κ. To select the poses we adopt the active calibration algorithm developed by (Sun and Hollerbach, 2008) that reduces the complexity of computing an observability index reducing computational time for finding optimal poses. The optimized identification set of postures is then defined by:

$$\begin{gathered} \mathbf{R}_\kappa : O_1(C_\kappa) \text{ is maximal.} \\ O_1(C_\kappa) = \frac{\sqrt[n_\kappa]{s_1\ s_2 \cdots s_{n_\kappa}}}{n_\kappa} \\ \mathbf{R}_\kappa \subset \mathbf{W_R} \\ \kappa = 1, 2, \ldots, n_{limbs} \end{gathered} \tag{7}$$

were O_1 is an observability index of the total identification matrix C_κ of the κth leg, n_κ is the number of parameters to be identified in each leg and, $s_1,\ s_2\ \ldots, s_{n_\kappa}$ are the singular values of the identification matrix C_κ. As a rule of thumb, in order to suppress the influence of measurement noise, the number of identification poses should be two or three times larger than the number of parameters to be estimated (Jang et al., 2001).

3. Given the optimized set of identification poses obtained in 2 and the inverse kinematic function (g_κ, Eq. 2) to solve the optimization problem defined on Eq. 5 for the identification of the kinematic parameters (φ_κ) of the κth leg.

4. Given the identified set of parameters of the κth leg obtained in 3 to update the kinematic model of the parallel mechanism.

The protocol is repeated until all the legs in the mechanism are identified.

With respect to traditional identification algorithms for the kinematic identification of parallel mechanism (Renaud et al., 2006; Zhuang et al., 1998) the proposed kinematic identification protocol has the following advantages:

1. Reduction of the kinematic identification computational costs. If a linear least-squares estimation of the kinematic parameters is used to solve the identification problem (Eq. 5), then the correction to be applied to the kinematic parameters

($\Delta\varphi$) can be estimated iteratively as (Hollerbach and Wampler, 1996):

$$\Delta\varphi = \left(C^T C\right)^{-1} C^T \Delta\mathbf{Q} \tag{8}$$

The computational cost of the matrix inversion $(C^T C)^{-1}$ is reduced proportionally to the square of the number of legs of the parallel mechanism, Table 1.

2. Improvement of the numerical efficiency of the kinematic identification algorithm by the independent identification of the parameters of each leg and,
3. Improvement of the kinematic identification by the design of independent experiments optimized for the identification of each leg.

Table 1: Computational and measurement costs of kinematic identification.

	Traditional kinematic identification	Divide and conquer identification
Regressor	$C^T C(N\, n_{limbs} \times N\, n_{limbs})$	$C_\kappa^T C_\kappa(N \times N)$
Computational cost (Matrix inversion)	$\propto N^3\, n_{limbs}{}^3$	$\propto N^3\, n_{limbs}$

The kinematic identification of parallel mechanisms protocol is applied in the simulated identification of a planar 5R symmetrical mechanism in section 5.

5 RESULTS

The results on kinematic identification of parallel mechanisms by a divide and conquer strategy are presented using a case study: the kinematic identification of the planar 5R symmetrical parallel mechanism.

The planar 5R symmetrical mechanism (Fig. 2) was proposed as a mean to overcome the reduced load-carrying capacity of planar two-degree-of-freedom serial-type manipulators (Cervantes-Sánchez et al., 2001). The mechanism has two degrees-of-freedom (DOF) that allows to positioning the end-effector point (P) in the plane that contains the mechanism. The mechanism is formed by two driving links (l_1 and l_2) and a conducted dyad (L_1 and L_2), Fig. 2. Several research works were developed for the planar 5R symmetrical mechanism. A complete characterization of the assembly configurations (Cervantes-Sánchez et al., 2000), kinematic design (Cervantes-Sánchez et al., 2001; Liu et al., 2006a; Liu et al., 2006b; Liu et al., 2006c), workspace (Cervantes-Sánchez et al., 2001; Cervantes-Sánchez et al., 2000; Liu et al., 2006a), singularities (Cervantes-Sánchez et al., 2001; Cervantes-Sánchez et al., 2000; Liu et al., 2006a) and performance atlases (Liu et al., 2006b) are reported. However, no research is reported on kinematic identification. The planar 5R symmetrical mechanism is an instance of parallel symmetrical mechanisms. Parallel symmetrical mechanisms are defined in section 3.

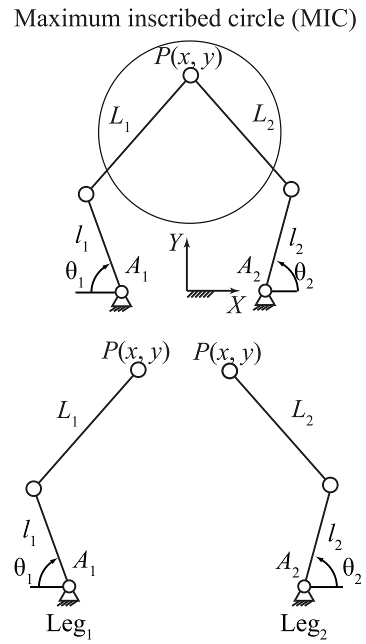

Figure 2: Planar 5R symmetrical mechanism

The kinematic identification of the planar 5R symmetrical mechanism is simulated using the kinematic identification of parallel symmetrical mechanisms protocol (section 4) under the following conditions:

1. A linear model is assumed for the active joints A_1 and A_2.

$$\begin{aligned} \theta_1 &= k_1\psi_1 + \gamma_1 \\ \theta_2 &= k_2(\pi - \psi_1) + \gamma_2 \end{aligned} \tag{9}$$

where the k_i represent the joint gain, γ_i is the joint offset, ψ_i is the commanded active joint angle and θ_i is the measured active joint angle.

2. In parallel mechanisms the principal source of error in positioning is due to limited knowledge of the joint centers, leg lengths and active joint parameters (Daney et al., 2002). In consequence, the parameters to be estimated are the attachment points (A_i), the leg lenghts (l_i, L_i) and, the joint

gain and offset (k_i, γ_i):

$$\begin{aligned}\varphi_1 &= [l_1\ L_1\ A_{1x}\ A_{1y}\ k_1\ \gamma_1]^T \\ \varphi_2 &= [l_2\ L_2\ A_{2x}\ A_{2y}\ k_2\ \gamma_2]^T \\ \varphi &= [\varphi_1\ \varphi_2]^T\end{aligned} \tag{10}$$

3. The external parameters associated with the measuring device will not be identified. For the external measuring system this implies that its position is known and coincident with the reference frame $X-Y$ and the measurement target is coincident with the end-effector point
4. The nominal kinematic parameters of the mechanism are disturbed adding a random error with normal distribution and a standard deviation σ. The nominal and disturbed parameters are shown in Table 2.

Table 2: Identification results.

		Nominal parameters	Disturbed (real) parameters
A_{1x}	[m]	-0.5000	-0.5057
A_{1y}	[m]	0.0000	0.0060
k_1		1.0000	1.0041
γ_1	[rad]	0.0000	$6.4{\cdot}10^{-04}$
l_1	[m]	0.7500	0.7478
L_1	[m]	1.1000	1.1006
A_{2x}	[m]	0.5000	0.5059
A_{2y}	[m]	0.0000	$-1.9{\cdot}10^{-04}$
l_2	[m]	0.7500	0.7417
L_2	[m]	1.1000	1.1014
k_2		1.0000	1.0045
γ_2	[rad]	0.0000	0.0046

5. The constrain equation of the inverse kinematics is defined to be, Fig. 3:

$$\mathbf{P} = \mathbf{A} + \mathbf{l} + \mathbf{L} \tag{11}$$

Developing Eq. 11 for each leg:

$$\begin{aligned}{L_1}^2 &= (x - l_1\cos\psi_1 - A_{1x})^2 + (y - l_1\sin\psi_1 - A_{1y})^2 \\ {L_2}^2 &= (x - l_1\cos\psi_2 - A_{2x})^2 + (y - l_1\sin\psi_2 - A_{2y})^2\end{aligned} \tag{12}$$

6. The end-effector and joint workspace are limited to the maximal inscribed workspace (MIW), Fig. 2. The MIW corresponds to the maximum singularity-free end-effector workspace limited by a circle (Liu et al., 2006c).
7. Linearization of the inverse kinematics is used for iteratively solving the non-linear optimization problem, Eq. 5. In terms of the independent identification of the kinematic parameters of a leg and

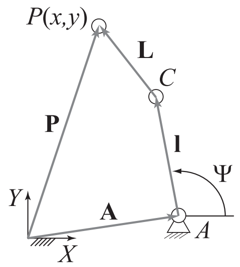

Figure 3: Planar 5R symmetrical mechanism. Leg loop.

for the jth identification pose the linearization has the form:

$$\begin{aligned}\Delta\mathbf{q}^j_\kappa &= \frac{\partial g^j_\kappa}{\partial\varphi_\kappa}\Delta\varphi_\kappa = C^j_\kappa\Delta\varphi \\ \Delta\mathbf{q}^j_\kappa &= \hat{\mathbf{q}}^j_\kappa - \bar{\mathbf{q}}^j_\kappa \\ \Delta\varphi_\kappa &= \hat{\varphi}_\kappa - \varphi_\kappa \\ \kappa &= 1,2\end{aligned} \tag{13}$$

Using N measurements to identify the set of parameters φ_κ the non-linear identification problem can be expressed as:

$$\begin{aligned}\Delta\mathbf{Q}_\kappa &= C_\kappa\Delta\varphi_\kappa \\ C_\kappa &= \left[C^1_\kappa \cdots C^N_\kappa\right]^T \\ \Delta\mathbf{Q}_\kappa &= \left[\Delta\mathbf{q}^1_\kappa \cdots \Delta\mathbf{q}^N_\kappa\right]^T \\ \kappa &= 1,2\end{aligned} \tag{14}$$

were C_κ is the total identification matrix of the κth leg. The parameters of the κth leg can be updated using a linear least-squares solution of Eq. 14, (Hollerbach and Wampler, 1996):

$$\Delta\varphi_\kappa = (C^T_\kappa C_\kappa)^{-1}C^T_\kappa\Delta\mathbf{Q}_\kappa \tag{15}$$

8. Each leg is identified using a set of 18 postures of the mechanism to measure the end-effector position and the corresponding active joint variable. The designed sets of identification postures in the end-effector workspace are presented in fig. 4b (left leg) and 4c (right leg).
9. The set of end-effector measurements ($\hat{\mathbf{R}}_\kappa$) and its corresponding active joint measurements ($\hat{\mathbf{Q}}_\kappa$) are simulated using forward kinematics and adding random disturbances with normal distribution and standard deviation $\sigma = 1\cdot10^{-4}$.
10. An alternative traditional kinematic identification by inverse kinematic modeling is calculated and used as a comparison to the divide and conquer strategy. The traditional identification is performed by a set of 36 optimized postures such

that:

$$\begin{aligned} \mathbf{R} : O_1(C) \text{ is maximal.} \\ O_1(C) = \mathbf{R} \subset MIW \end{aligned} \tag{16}$$

were O_1 is the observability index of the total identification matrix C of the total set of parameters of the mechanism. The index O_1 is defined as in the Eq. 7. The designed set of identification postures in the end-effector workspace is presented in fig. 4a.

The result of the kinematic identification under these conditions are presented in Fig 4, (selected postures for kinematic identification), Fig. 5 (residual errors in kinematic parameters before and after calibration), these residual errors are calculated as the difference between the real (virtually disturbed) parameters and the estimated parameters. Finally, Fig. 6 presents the estimated local root mean square error for MIW after calibration. Additionally the computational and measurement identification costs are estimated for the identification of the planar 5R parallel mechanism, Table 3. The measurement costs of the Divide and Conquer strategy are incremented with respect to a traditional identification method 3. The increment of the measurements is required for the independent identification of the legs: each leg requires an independent set of end-effector measurements. In the case of a traditional identification the set of end-effector measurements is common to all the legs. In despite of the measurement increment the Divide and Conquer identification results in a superior estimation with respect to a traditional kinematic identification methods (Renaud et al., 2006; Zhuang et al., 1998).

The conclusions of the paper are proposed in section 6.

Table 3: 5R parallel mechanisms. Computational and measurement costs of kinematic identification.

	Traditional kinematic identification	Divide and conquer identification
Regressor	$C^T C(36 \times 36)$	$C_\kappa^T C_\kappa(18 \times 18)$
Computational cost (Matrix inversion)	$\propto 18^3 \cdot 2^3$	$\propto 18^3 \cdot 2$
Measurement cost	$2 \cdot 18 \cdot 2 = 72$	$18 \cdot 2(2+1) = 108$

6 CONCLUSION

This article presents a new (Divide and Conquer) strategy for the kinematic identification of parallel symmetrical mechanisms. The new strategy develops a formalization of the inverse calibration method proposed by (Zhuang et al., 1998). The identification strategy (section 3) is based on the independent identification of the kinematic parameters of each leg of the parallel mechanism by minimizing an error between the measured active joint variable of the identified leg and their corresponding value, estimated through an inverse kinematic model. With respect to traditional identification methods the Divide and Conquer strategy presents the following advantages:

1. Reduction of the kinematic identification computational costs,
2. Improvement of the numerical efficiency of the kinematic identification algorithm and,
3. Improvement of the kinematic identification results.

Based on the Divide and Conquer strategy, a new protocol for the kinematic identification of parallel symmetrical mechanisms is proposed (section 4, Fig. 1). For the selection of optimal identification postures the protocol adopts the active robot calibration algorithm of (Sun and Hollerbach, 2008). The main advantage of the active robot calibration algorithm is the reduction of the complexity of computing an observability index for the kinematic identification, allowing to afford more candidate poses in the optimal pose selection search. The kinematic identification protocol summarizes the advantages of the Divide and Conquer identification strategy and the advantages of the active robot calibration algorithm.

The kinematic identification protocol is demonstrated with the (virtual) identification of a planar 5R symmetrical mechanism (section 5). The performance of our identification protocol is compared with a traditional identification method obtaining an improvement of the identification results (Figs. 5 and 6).

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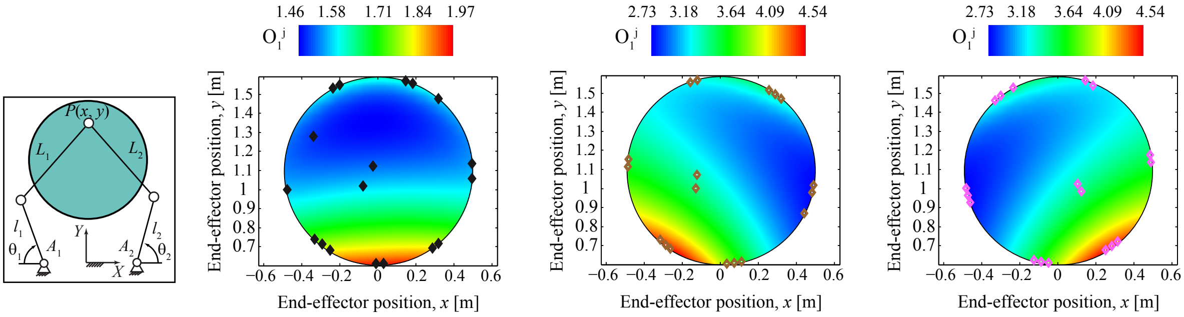
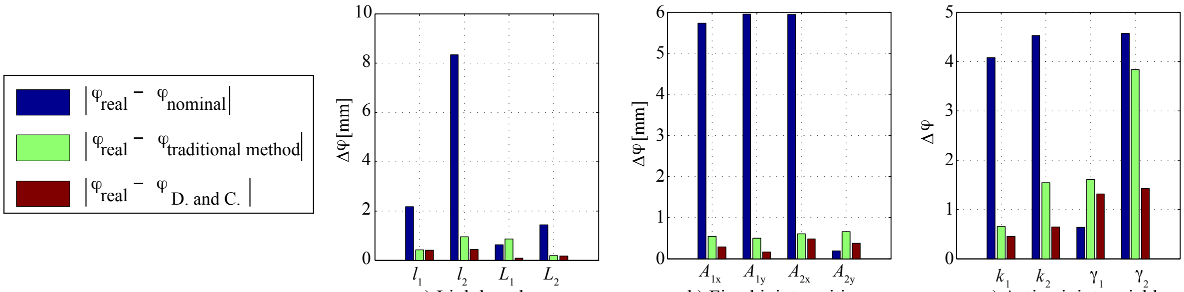


a) Optimal postures for traditional ident. method

b) Optimal postures for D&C method. Left leg

c) Optimal postures for D&C method. Right leg

Figure 4: Planar 5R mechanism. Selected postures for kinematic identification.

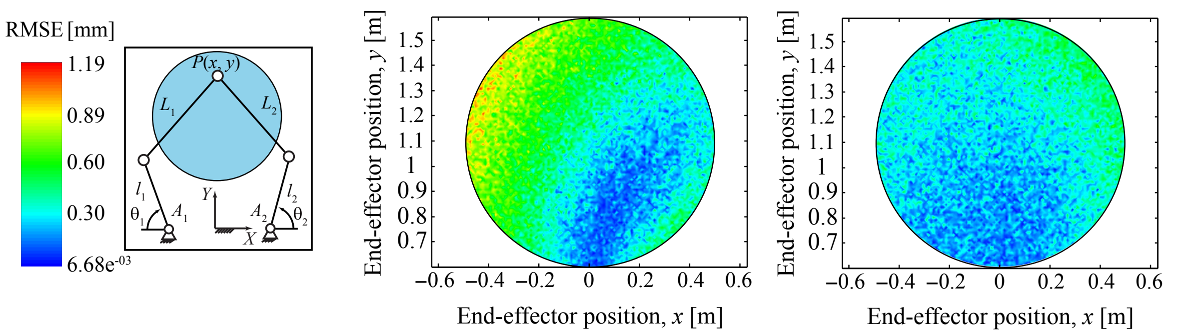


a) Link lengths

b) Fixed joint positions

c) Active joint variables

Figure 5: Planar 5R mechanism. Residual errors in the kinematic parameters before and after calibration.



a) Calibration with traditional identification Method

b) Calibration with Divide and conquer identification strategy

Figure 6: Planar 5R mechanism. Estimated end-effector local root mean square error for the maximal inscribed workspace (MIW) after calibration.

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