Analytical Calculation of Friction Forces in Assur Groups.

S. Durango [a,*],G. Calle [b],O. Ruiz [c]

[a] *Mechanics Eng. Department, Universidad Autónoma de Manizales, Antigua estación del ferrocarril, Manizales, Colombia.*
[b] *Mechanics Eng. Faculty, Universidad Tecnológica de Pereira, vereda La Julita, Pereira, Colombia.*
[c] *CAD/CAM/CAE laboratory, EAFIT University, Carrera 49 No 7 sur - 50, Medellín, Colombia.*

Abstract

The calculation of forces in the kinematical pairs of articulated mechanisms is usually performed without friction force considerations. In practice, when examination of articulated mechanisms takes into account friction, the solution of this problem results very complex. Exact solutions are available, but not necessarily are practical. For example, the analytical solution for a second-class first-type Assur group is a 16th degree equation. Previous investigations proposed an approximated but practical (graphical) method to determine the forces on the kinematical pairs taking into account the friction forces. In this article, an analytical interpretation of the Artobolevski approximated method is developed for second-class mechanisms. The final results for the reactions calculated with the implemented method present a good approximation with respect to the graphical solution. In the particular case of mechanisms in configurations near to self-brake the method is not accurate. Future work should consider friction forces not only in second-class but in superior-class Assur groups.

Key words:
Friction, Assur group, second-class mechanisms, kinetostatics

Nomenclature

R - Rotational kinematical pair
P - Prismatic kinematical pair
F_f - Friction force magnitude
f - Dry friction coefficient
M_f - Friction moment in a rotational pair
r - shaft radius in a rotational pair
$\mathbf{R}_{PQ}$ - Relative vector position of P with respect to Q
F_i - Sum of external and inertial forces over link i
$M_{ij,f}$ - Pair friction moment of link i over link j
$\mathrm{sgn}()$ - Sign function
F_{ij}^t - Pair reaction, effect of link i over link j, component perpendicular to the link j
T_c - Drive input torque for the driving mechanism
γ, δ - Geometrical angular parameters
l - Geometrical length parameter
F - Force magnitude
M - Moment of a force, force pair
F_n - Normal to surface force magnitude
φ - Friction angle
ρ - Friction circle radius in a rotational pair
f' - Friction coefficient for a rotational pair
F_{ij} - Pair reaction, effect of link i over link j
$M_{Fi,Q}$ - Sum of moments of external and inertial forces over link i with respect to Q
M_i - Sum of external and inertial moments over link i
ω_i - Angular velocity of link i
F_{ij}^n - Pair reaction, effect of link i over link j. component in the direction of link j
m_i - Mass of link i
J_i - Inertial moment of mass of link i

* Corresponding Author, Tel. +57-4-261-9500 ext 609
Email addresses: sebastiandi@autonoma.edu.co (S. Durango), gcalle@utp.edu.co (G. Calle), oruiz@eafit.edu.co (O. Ruiz).

1. Introduction

In the context of the kinematical pair force calculation of the articulated mechanism is usual to avoid friction effects. Initially, the calculation of friction forces in these mechanisms has no difficulties. The reaction in a translational pair must be in the direction of the friction angle with respect to the normal, and the reaction in a rotational pair must be tangential to the friction circle. However, with respect to the kinematical pair force including friction calculations for articulated mechanisms [25] concludes that the analytical solution of this problem results extraordinarily complex. The problem complexity is due to the non-linear character of the problem. A graphical iterative solution to the problem of the calculation of forces in kinematical pairs taking into account friction is proposed in [24].

This paper presents a practical alternative to the calculation of forces in kinematical pairs in second-class mechanisms taking into account the effects of friction. The proposed approach corresponds to the so called general purpose programs (GP) based in kinematical units. Major advantages of chain based GP are flexibility and computer efficiency, being possible to analyze a great variety of mechanisms. To develop a chain based GP is necessary to obtain independent analytical solutions for each structural group. Once such a solution is obtained it can be codified as an independent module of a library.

A methodology to obtain a kinematical pair force model with friction effects for any structural group is proposed. The methodology includes computer aided graphical validations. The solution of the second-class first type Assur group is developed as an application example.

2. Literature review

2.1. *Computer aided analysis methods for mechanical systems*

Computer aided analysis methods for mechanical systems are divided by [23] into two categories:

(i) Special Purpose (SP)
(ii) General Purpose (GP)

Special purpose methods are typically limited to a specific mechanism. Some examples of SP programs are: *Fourbar*, *Slider*, and *Dynacam* designed by Norton [22] for the kinematics and dynamic analysis of four-bar mechanisms, crank-slider mechanisms and cam mechanisms.

General-purpose methods are codified as libraries without any specific mechanism, but including the necessary elements to virtually assembly a mechanism. General purpose libraries could be developed over the kinematic joint concept or over the kinematic unit concept.

For joint based programs the library contains a number of kinematic joints. Reference [11] propose the principal advantage of this approach as the effectiveness to define a complex spatial problem. Typical examples of joint based programs are *Working Model*, *Adams* and *Simmechanics*.

Libraries developed using the kinematic unit approach contain a number of kinematic chains with a special characteristic that can be assembled in some way to form a mechanism. The most important feature of this kind of programs is that it comprises the advantages of both SP programs and joint based GP programs, [11] lists them as: computational efficiency and flexibility.

Some references about academic and commercial kinematic unit based programs are [21], [11], [13] [14], [15], [20], but there are not friction considerations for the kinematical pairs in those programs. An alternative computer aided solution for the kinematics and dynamic analysis of mechanisms consists of graphical solutions using parametric CAD software. Ref. [18] presents an approximation for this type of analysis, but it could not be practical for complex mechanisms.

The present paper propose an analytical, but practical method to estimate kinematical pair forces including friction effects in structural groups. The analytical and modular form of the solutions allows to develop a general purpose library based in the kinematic unit approach.

2.2. *Structural classification of mechanisms*

In 1916 scientist L. V. Assur [16] proposed a new method to classify mechanisms based on the concept of structural groups that corresponds to the kinematic unit concept. One accepted definition of structural group is:

Def. **Structural group**. It is a kinematic chain with no degrees of freedom with respect to the links with which its free elements form pairs. Such a chain cannot be divided into simpler kinematic chains with the same characteristic.

And for the driving input mechanism:

Def. **first class mechanism**. It consists of the driving input and the fixed link forming a one degree of freedom pair.

The formation of any planar mechanism can be understood as the serial union of structural groups and one or more first class mechanisms. Consider for example the mechanism in Fig. 1, the structural law of formation is as follows: the mechanism has one degree of freedom with a first class rotational mechanism formed by the fixed link and link *1*. A structural group formed by links *2* and *3* is added between link *1* and the fixed link. Finally a structural group formed by links *4* and *5* is added between links *2* and *3*. The formation law is:

$$I\,R_{0,1} \rightarrow II\,RRR_{2,3} \rightarrow II\,RRP4,5 \tag{1}$$

In Eq. (1) R denotes a rotational pair and P a prismatic pair.

A complete review on structural classification of mechanisms until 2003 is presented in [4]. Novel aspects and computational strategies to determine the structural classification of a planar mechanism are developed in [1] and [2]. For more information on structural classification see [1], [2], [4], [10], [21], [24] and [25].

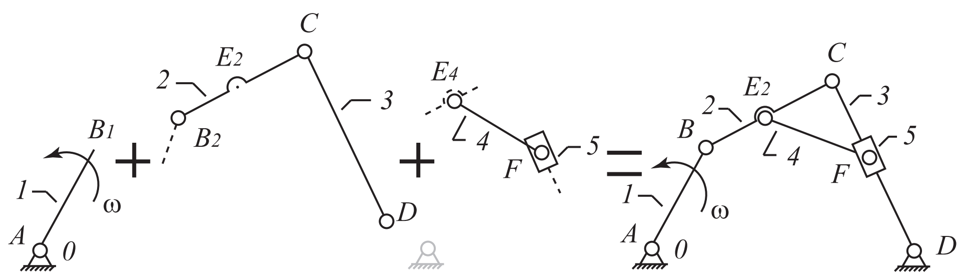

Fig. 1. Second-class mechanism. Structural synthesis

2.3. *Rotational pair friction simplified model*

A common and simplified model of the dry friction acting on rotational pairs results of considering a shaft over its support Fig. (2).

The shaft *1* is over the support *2* under the action of a radial force $\mathbf{F}$ and an external moment M. The shaft is assumed rotating with constant angular velocity ω. There is a radial tolerance between the shaft and its support. When the shaft rotates in the indicated direction it tends to ascend over the support because of friction. Assuming that the contact point moves to A the reaction $\mathbf{F}^*$ appears parallel to the radial force $\mathbf{F}$. The normal component of the reaction force (F_n) is inclined by an angle φ and the magnitude of the friction force (F_f) is:

$$F_f = fF_n = fF^* \cos\varphi = fF\cos\varphi \tag{2}$$

Where f is the friction coefficient between the materials on the pair.

The moment M applied to the shaft is equilibrated whit the friction moment M_f, which is equal to:

$$M_f = F_f r = fFr\cos\varphi = Fr\sin\varphi = F\rho \tag{3}$$

Where $\rho = r\sin\varphi$. The friction angle φ is usually small, then the approximation $\sin\varphi \cong \tan\varphi$ holds and the radius ρ is approximated by:

$$\rho \cong fr \tag{4}$$

The friction moment on the rotational pair is generally determinated by:

$$M_f = Frf' \tag{5}$$

Where r is the radius of cylindrical element on the pair, f' is the friction coefficient on the rotational pair and F is the resultant load over the shaft.

The friction coefficient f' is determined experimentally for different work conditions of the kinematical pair and changes depending of the materials, the state of friction surfaces and the conditions of work. For dry friction on wear less pairs generally $f' = \frac{3}{2}f$ and for worn out pairs $f' = \frac{4}{3}f$.

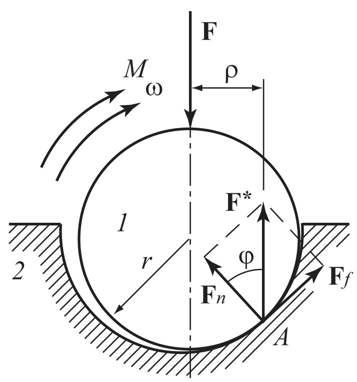

Fig. 2. Shaft over its support ([24])

2.4. *Analytical calculation of forces in second-class structural groups*

Methods of kinematic and dynamic analysis based on structural classification are general and modular. The analysis of a structural group can be established independently and then codified as a computer function or module of a library. A structural group can be classified with respect to the number of links and type of pairs that forms it. Second-class structural groups are formed by 2 links and 3 pairs of one degree of freedom (Fig. 3). The variety of mechanisms that could be analyzed with a library depends of the number of modules that the library contains. However most of industrial mechanisms are formed by combinations of driving mechanisms and second-class structural groups.

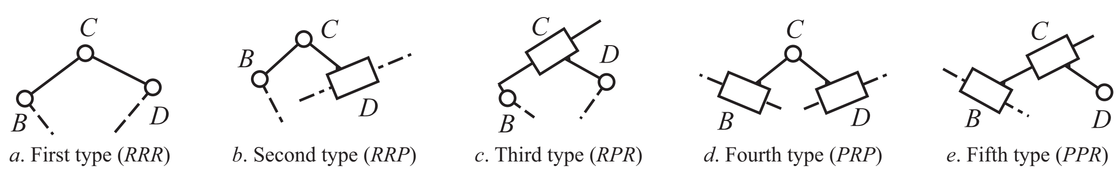

Fig. 3. Second-class structural groups

2.4.1. *Analytical methods of analysis*

Analytical methods of analysis for planar mechanisms based on the structural classification can be seen on [2], [3], [5], [6], [7], [8], [9], [12] and [19] for kinematics and [11], [13], [14], [15], [17] for dynamics. There is no friction considerations on these works.

The kinematical analysis of a mechanism with a given structural classification is solved in the direction of the structural law. However the kinetostatics analysis is solved in the opposite way of the law of formation.

In kinematics, the law of movement for the primary mechanisms is given. Therefore it is possible to stream the information from group to group until the entire mechanism is solved (Fig. 4).

For the solution of the kinetostatics is possible to use the D'Alembert principle because each structural group is statically determined. It is then is possible to isolate the last group in the structural law to solve its reactions. The solution comes from group to group until the driving force or moment was solved (Fig. 4).

The analytical solution of the kinetostatics of structural groups without friction effects results in a linear problem. However when kinematical pair friction is taking into account the problem comes non linear and complex. In particular for a second-class first-type structural group an exact analytical solution with friction calculations results in a 16-degree equation ([25]). Therefore a simpler estimation method is needed.

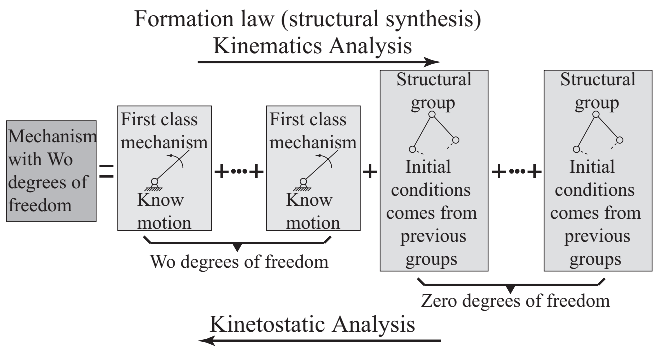

Fig. 4. Kinematics and kinetostatics analysis aspects.

2.4.2. *Graphical methods of analysis*

Artobolevski [24] proposed a graphical approximated method for the calculation of reactive forces in kinematical pairs that includes friction. The calculation of reactions starts without the inclusion of friction forces. When the reactions without friction (F_n) are know it is possible to find the associated friction forces (F_f). The friction forces are orientated perpendicularly to the original reactions (Fig 5). Friction forces can be calculated as:

$$F_{12f} = -F_{21f} = f' F_n \tag{6}$$

Once friction forces are calculated they can be applied to the corresponding elements. The force $\mathbf{F}_{21,f}$ is applied to the element *1* and $\mathbf{F}_{12,f}$ to the element *2*. The direction of the friction forces is opposite to the relative angular velocity $\omega_{12} = \omega_1 - \omega_2$ and $\omega_{21} = \omega_2 - \omega_1$.

After this, the conventional method of force calculation is resumed. The new values of the reactions calculated by this way generally has little difference with respect to those calculated by an exact solution ([25]).

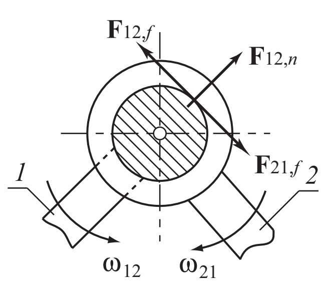

Fig. 5. Friction forces in a rotational pair ([24])

It is possible to apply the previously mentioned method to different structural groups avoiding non-linearities and with sufficient precision.

Ref. [25] recommends not to apply this method when the analyzed mechanism is near to self-brake. In such configurations the influence of friction forces is major and the approximation could not be valid.

3. Methodology

To obtain an iterative analytical solution to the kinematical pair forces in structural groups including friction effects the methodology presented in Fig. 6 is proposed:

(i) Obtain a D'Alembert formulation without friction forces and moments for the structural group.
(ii) Use the D'Alembert formulation to obtain an analytical solution for forces in pairs of the structural group
(iii) Establish a friction model for the kinematical pairs in the structural group.
(iv) Include the friction forces and moments in the D'Alembert formulation.
(v) Codify independent computer function with successive approximation process for pair force calculation in the structural group.
(vi) Develop a parametric computer aided drawing (CAD) validation for pair forces in the structural group.
(vii) Solve application problems

As an example of the proposed methodology, the solutions for the second-class first-type structural group and rotational primary mechanism are developed in sections 3.1 and 3.2. This methodology is expandable to other second-class or superior Assur groups.

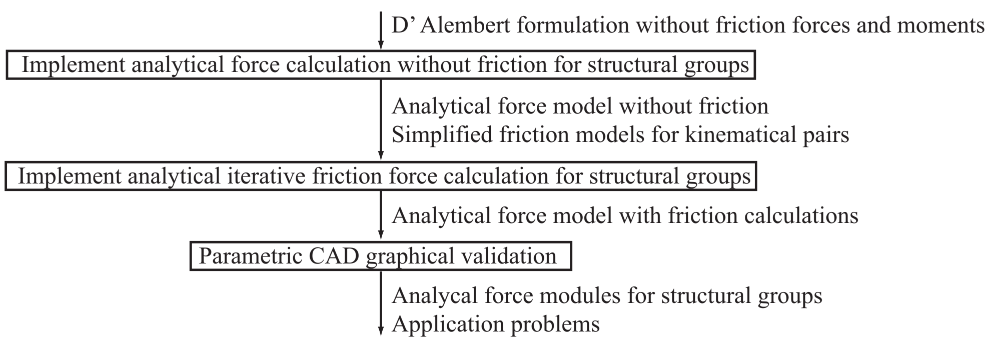

Fig. 6. Proposed methodology for structural group modeling with friction force calculations

3.1. *Kinetostatics of the second-class first-type Assur group, analytical formulation*

For the analytical study of the kinetostatics of a second-class first-type Assur group are known parameters: links geometry, kinematics and inertial characteristics. Fig. 7 presents a dynamic equilibrium diagram of the

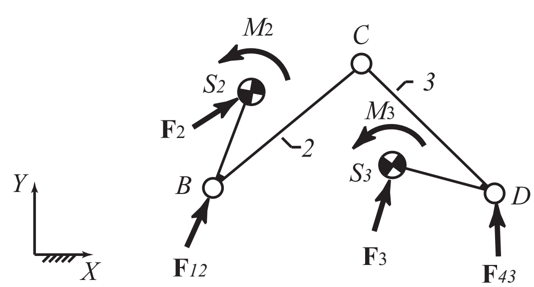

Fig. 7. Second-class first-type Assur group, dynamic equilibrium diagram

group. Solving the sum of forces over the entire group by the D'Alembert principle:

$$F_{12x} + F_{2x} + F_{3x} + F_{43x} = 0 \tag{7}$$

$$F_{12y} + F_{2y} + F_{3y} + F_{43y} = 0 \tag{8}$$

Where F_{ij} corresponds to the force of link i over link j, F_2, F_3 is the sum of the external and inertial forces over links *2* and *3* correspondently.

Taking moments around C, for links *2* and *3*:

$$R_{BCx}F_{12y} - R_{BCy}F_{12x} + M_2 + M_{F2,C} = 0 \tag{9}$$

$$R_{DCx}F_{43y} - R_{DCy}F_{43x} + M_3 + M_{F3,C} = 0 \tag{10}$$

Where M_i is the sum of the external and inertial moments over link i and $M_{Fi,C}$ is the sum of the moments of external and inertial forces of link i with respect to C.

Eqs. (7), (8), (9) and (10) forms a linearly independent system for the reactions $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$ without friction. To calculate the reaction on the C pair the sum of forces over link *2* or *3* can be solved. Taking the sum of forces over link *2*:

$$F_{12x} + F_{2x} + F_{32x} = 0 \tag{11}$$

$$F_{12y} + F_{2y} + F_{32y} = 0 \tag{12}$$

To estimate the kinematical pair reactions with friction effects is necessary to calculate the friction moments $M_{12,f}$, $M_{23,f}$, $M_{32,f}$ and $M_{43,f}$. Considering the eq. (5), and including the direction of the friction moments using the sign function:

$$M_{12,f} = -\mathrm{sgn}(\omega_1 - \omega_2)\, F_{12} f'_B r_B \tag{13}$$

$$M_{32,f} = -M_{23,f} = -\mathrm{sgn}(\omega_3 - \omega_2)\, F_{32} f'_C r_C \tag{14}$$

$$M_{43,f} = -\mathrm{sgn}(\omega_4 - \omega_3)\, F_{43} f'_D r_D \tag{15}$$

Where ω_i is the angular velocity of the link i, f'_i is the friction coefficient in the rotational pair i and r_i is the radius of the cylindrical element on the pair. Including the friction moments on the D'Alembert formulation eq. (9) and eq. (10) (Fig. 8):

$$R_{BCx}F_{12y} - R_{BCy}F_{12x} + M_2 + M_{F2,C} + M_{12,f} + M_{32,f} = 0 \tag{16}$$

$$R_{DCx}F_{43y} - R_{DCy}F_{43x} + M_3 + M_{F3,C} + M_{23,f} + M_{43,f} = 0 \tag{17}$$

In Eq. (16) and Eq. (17) friction moments are functions of the magnitude of the radial forces on each pair. Therefore is not possible to determine directly the reactions $\mathbf{F}_{12}$, $\mathbf{F}_{32}$, and $\mathbf{F}_{43}$ by means of the set of expresions (7), (8), (16), (17), (11) and (12). A practical form to solve this problem is to use a method of successive approximations that is described forward.

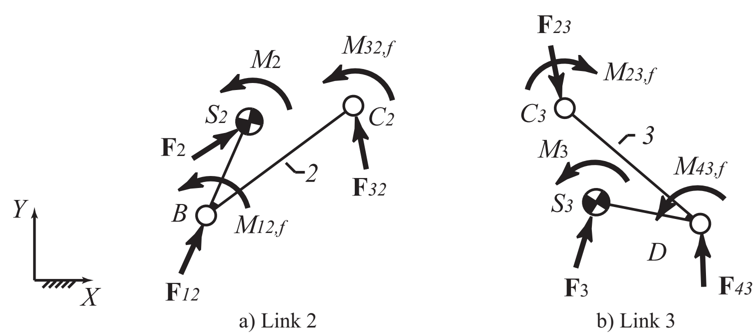

Fig. 8. Second-class first-type Assur group, dynamic equilibrium diagrams with friction moments

(i) For the first approximation the moments of the friction forces are equal to zero. $M_{12,f} = 0, M_{32,f} = -M_{23,f} = 0$, and $M_{43,f} = 0$. The problem is reduced to the solution of the kinetostatics without friction by the linear set of Eqs. (7), (8), (9), (10), (11) and (12).
(ii) The initial estimation of the friction moments is calculated by the Eqs. (13), (14) and (15).
(iii) Including the friction moments in Eqs. (11) and (12) a new set of reactions $\mathbf{F}_{12}, \mathbf{F}_{32}, \mathbf{F}_{43}$ are determined.
(iv) To iterate the calculation process the new values for F_{12}, F_{32}, F_{43} are used in step (ii) for friction moments. A new set of kinematical pair reactions is determinated in step (iii).
(v) A criteria for breaking the iteration process could be the difference of the magnitude of the reactions on consecutive calculations.

Fig.9 presents a diagram for the proposed model.

The obtained model (Fig. 9) for the second-class first-type Assur group can be codified as an independent module or computer function. Such a function could be included in a GP chain based library. To analyze a particular mechanism is necessary that each structural group in its structural law was included in the library.

The method of solution presented here for the kinetostatics analysis of the second-class first-type Assur group is applicable to any other second-class or superior-class Assur group. Also is possible to consider a more complex friction model for the pairs and include it on the method.

Once the model correspondent to a structural group is obtained a graphical validation is convenient. Section 3.1.1 presents a graphical method of analysis used to validate the analytical model for the second-class first-type structural group. The graphical method could be extend to other second-class or superior-class structural groups.

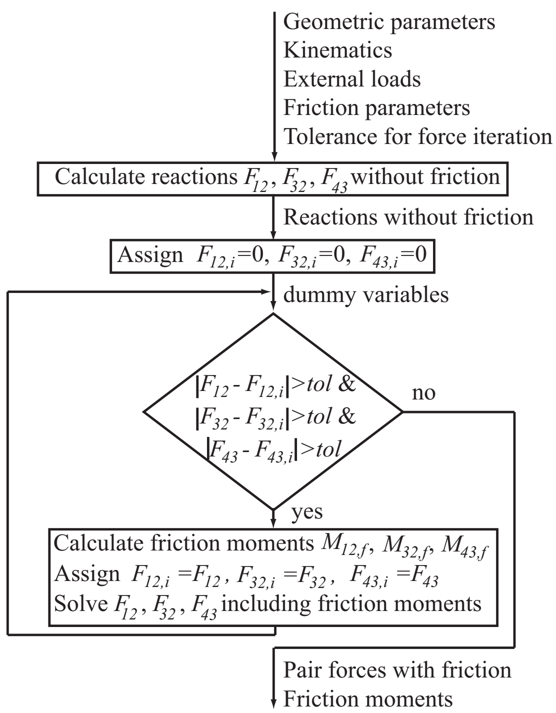

Fig. 9. Second-class first-type Assur group: diagram for kinetostatics with friction moments calculation

3.1.1. *Graphical method of analysis for validation*

In theory of machines and mechanisms graphical methods of analysis are considered to be both, classical and fundamentally valid. A graphical validation of the obtained model for the kinetostatics of the second-class firs-type Assur group is developed forward.

The graphical solution procedure described by [24] is show in Fig. 10. The kinematical pair forces are decomposed in parallel to link (F^n) and perpendicular to link (F^t) components.

(i) The first approximation consists in a standard graphical solution of the kinematical pair forces without friction effects. The initial reactions are labeled as $\mathbf{F}^o_{12}$, $\mathbf{F}^o_{23}$, $\mathbf{F}^o_{43}$.

(ii) Calculate initial values of the kinematical pair friction moments.

$$M'_{12f} = F^o_{12} f'_B r_B, \quad M'_{23f} = F^o_{23} f'_C r_C, \quad M'_{43f} = F^o_{43} f'_B r_B \tag{18}$$

(iii) Include friction moments in the D'Alembert formulation of the structural group (Fig. 8).

(iv) Solve the kinematical pair reactions in its normal and tangent to link components: $(F^t_{12})'$, $(F^n_{12})'$, $(F^t_{32})'$, $(F^n_{32})'$, $(F^t_{43})'$ and $(F^n_{43})'$ (Fig.10). The new reactions are different from the first ones. If in the first step the vector of F^t_{43} was on point d_o, now is in a different point d_1 (Fig.10). In the same way the vector F^t_{12} change from point e_o to point e_1 (Fig.10). These new values determine a second plane for the forces $\mathbf{F}'_{12}$, $\mathbf{F}'_{23}$, $\mathbf{F}'_{43}$.

(v) For more precision in the magnitudes of the pair reactions is possible to repeat the calculations. However with only 2 iterations reasonable results are obtained: $(F^t_{12})''$, $(F^n_{12})''$, $(F^t_{32})''$, $(F^n_{32})''$, $(F^t_{43})''$ and $(F^n_{43})''$ (Fig. 10).

This method can be applied to any class groups with rotational or translational pairs. Such a method is of successive approximations. It can be used in cases in which there are a convergence process.

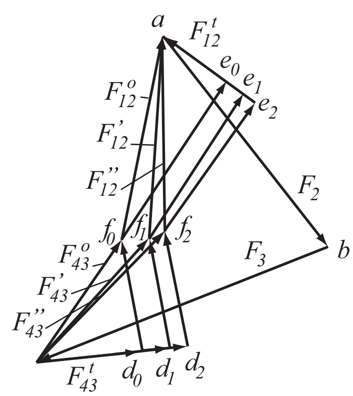

Fig. 10. Second-class first type Assur group: Forces plane with approximated friction calculations ([24]).

3.2. *Kinetostatics of the rotational primary mechanism.*

Fig. 11 presents the free body diagram for a rotational primary mechanism with a torque input and friction moments. The sum of forces for dynamic equilibrium is:

$$\mathbf{F}_{01} + \mathbf{F}_1 + \mathbf{F}_{21} = 0 \tag{19}$$

Where $\mathbf{F}_1$ is the sum of the external and inertial forces over link *1* and $\mathbf{F}_{ij}$ is the force of link i over link j. The force $\mathbf{F}_{21} = -\mathbf{F}_{12}$ comes from the solution of the next group in the structural law.

It is possible to solve the reaction $\mathbf{F}_{01}$ using directly the Eq. (19). Once the reaction is solved it is possible to determine the input torque. Calculating the sum of moments with respect to A:

$$T_c + M_{21,f} + M_{01,f} + M_1 + M_{F1,A} + R_{BAx}F_{21y} - R_{BAy}F_{21x} = 0 \tag{20}$$

Where T_c is the required input torque necessary to dynamically equilibrate the mechanism, M_1 is the sum of the external and inertial moments and $M_{F1,A}$ is the sum of the moments of the inertial and external forces with respect to A.

The friction moments $M_{01,f}$ and $M_{21,f}$ can be determined directly by the Eq. (5). Including a sign function to the direction of moments:

$$M_{21,f} = -M_{12,f} = -\text{sgn}(\omega_2 - \omega_1)\, F_{21} f'_B r_B \tag{21}$$

$$M_{01,f} = -\text{sgn}(\omega_0 - \omega_1)\, F_{01} f'_A r_A \tag{22}$$

Where ω_i is the angular velocity of the link i, f'_i is the friction coefficient on the pair and r_i is the radius of the cylindrical element on the pair. The input torque (T_c) can be solved directly from Eq. (20).

The Eqs. (19), (20), (21) and (22) can be codified as an independent module or computer function. Such a function could be included in general purpose kinematic chain based library.

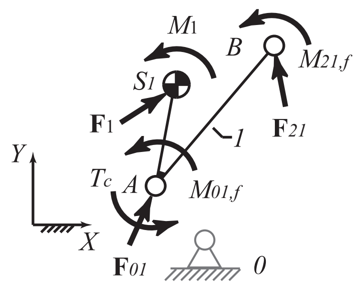

Fig. 11. Rotational primary mechanism: dynamic equilibrium diagram

4. Results

In this paper analytical friction force calculations are included in the kinetics solutions of second-class structural groups. An approximated but practical iterative method was developed to avoid non-linearities and complicated solutions.

For friction forces and moments in the kinematical pairs a simplified dry friction model was used as section 2.3 presents. However a more complete friction model could be used.

The presented method to estimate friction forces in kinematical pairs of structural groups consists in the calculation of friction forces and moments with successive approximations. This method can be used in cases in which there are a convergence process. Although the obtained solutions are specific for second-class structural groups it is possible to extend the presented method to superior-class structural groups.

To obtain analytical models for kinematical pair forces with friction effects in structural groups a methodology was proposed (Fig. 6). Analytical models for second-class structural groups an driving mechanisms were codified as user defined functions of MatLab. The set of modules (user defined functions) forms a library for the analysis of second-class mechanisms called *Assur Toolbox*. An alternative and more intuitive method to implement the code of each module is to use SIMULINK user defined masked blocks. In that case each structural group is represented by a block. Therefore the structural law of the mechanism can be established wiring properly the blocks (Fig. 12).

A computer aided graphical validation of the presented modules was developed using a parametric CAD software. As a validation model a second-class mechanism including a rotational driving mechanism and a second-class first-type structural group was used. Although parametric CAD solutions permits to evaluate several configurations of a mechanism can be considerate as an special purpose program.

As an analysis and validation problem a complete study of the kinetostatics of a four-bar mechanism is presented in sections 4.1 and 4.2. The validation was developed by a computer aided graphical method using the methodology of section 3.1.1 and the work presented in [18]. The proposed four-bar mechanism includes a second-class first-type mechanism and a rotational driving input mechanism.

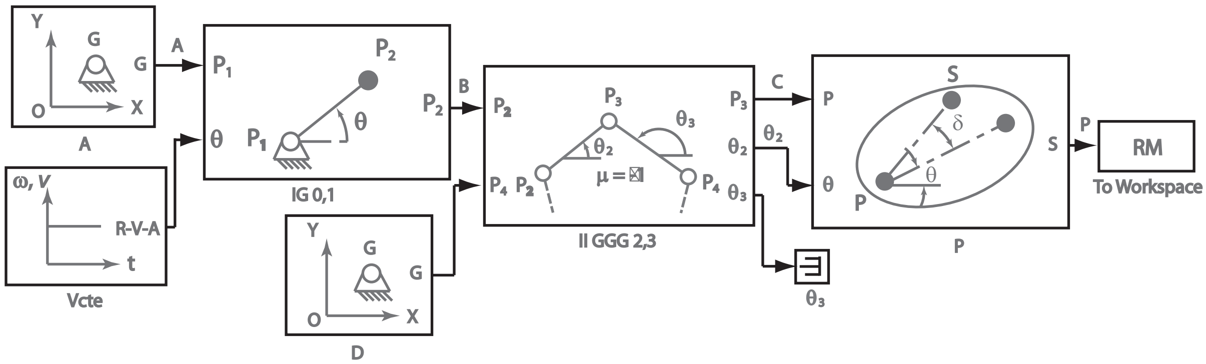

Fig. 12. Kinematical modular analysis of a four-bar mechanism using *Assur Toolbox*

4.1. *Analysis and validation problem*

Fig. 13 presents a four-bar mechanism specified in Table 1. For the analysis, the mechanism is consider to be in steady state. The input angular velocity is $\omega_1 = 10\,\text{rad/s}$.

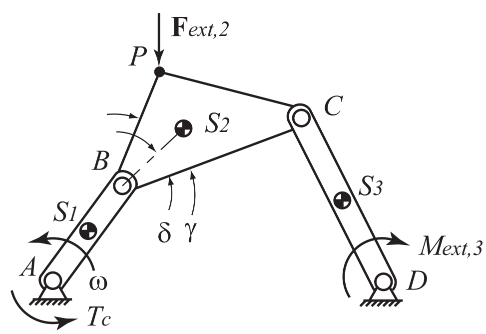

Fig. 13. Four-bar mechanism

Table 1
Four-bar mechanism: geometric and inertial parameters

Parameter	Magnitude	Parameter	Magnitude	Parameter	Magnitude
l_{AB}	$2,00\,\text{m}$	m_1	$4,8\,\text{kg}$	J_1	$1,70\,\text{kg}\,\text{m}^2$
l_{BC}	$6,00\,\text{m}$	m_2	$30,0\,\text{kg}$	J_2	$48,80\,\text{kg}\,\text{m}^2$
l_{CD}	$3,00\,\text{m}$	m_3	$7,2\,\text{kg}$	J_3	$5,50\,\text{kg}\,\text{m}^2$
l_{AD}	$5,50\,\text{m}$	$\mathbf{F}_{ext,2}$	$1,0\,\text{kN}\downarrow$	$M_{ext,4}$	$-250\,\text{N}\,\text{m}$
l_{BS2}	$2,80\,\text{m}$	δ	$0,152\,\text{rad}$	$r^{(a)}$	$5,0\,\text{mm}$
l_{BP2}	$2,50\,\text{m}$	γ	$\pi/6\,\text{rad}$	$f^{(a)}$	$0,25$

(a) Same radius and friction coefficient for all joints

The structural classification of the mechanism is $IR_{0,1} \rightarrow IIRRR_{2,3}$. To the analysis of the mechanism is necessary to use only two modules. One module corresponds to the rotational primary mechanism and one to the second-class structural group.

For the analysis of the four-bar mechanism we assume its geometry, kinematics, inertial parameters and structural classification to be known. The analysis process is as follows:

(i) Solve the second-class structural group kinematical pair reactions using the correspondent module in the library.

(ii) Stream the solutions to the driving input mechanism and solve the input drive torque and reactions. Fig. 14 shows the analysis process.

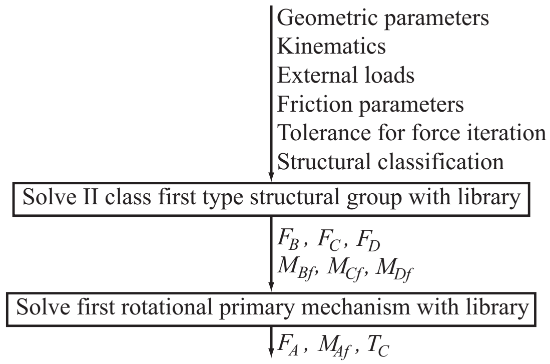

Fig. 14. Four-bar mechanism: Analysis process

The results of the analysis are presented in Fig. 15. There are comparisons between the proposed analytical approximated model and a solution with a joint based general purpose program. The comparison program uses a rough approximated solution without an iteration process. The presented outputs are: the required input torque, the friction power consumption and the reaction at pair D. In Fig. 15 the results are presented with solid lines for the proposed model and dashed lines for the compared solution. The criteria for breaking the iteration process was established in $1 \times 10^{(-3)}$ N for the difference on consecutive calculations of the magnitude of the pair reactions in the structural group.

4.2. *Computer aided graphical validation*

A computer aided graphical solution using a parametric CAD software was developed as a validation of the second-class first-type structural group model. Such a validation uses the method presented in section 3.1.1 with one iteration. The results for kinematical pair reactions B and D for several configurations of the mechanism are presented in Table 2.

Table 2
Four-bar mechanism: Parametric CAD graphical validation

	Analytical model				Graphical solution				
θ_1	F_A [N]	F_B [N]	F_C [N]	F_D [N]	F_A [N]	F_B [N]	F_C [N]	F_D [N]	max diff. %
0	27 235,7	26 766,2	18 364,2	17 940,6	27 232,6	26 763,2	18 360,6	17 936,9	0,020
30	11 117,6	10 676,2	4 843,8	4 631,8	11 117,7	10 676,3	4 843,9	4 632,0	0,003
90	2 776,6	2 296,6	1 944,6	2 458,5	2 776,6	2 296,6	1 944,6	2 458,5	0,001
135	3 689,5	3 278,0	1 604,6	1 959,9	3 689,6	3 278,1	1 604,6	1 959,9	0,005
225	6 478,1	5 999,1	244,3	252,7	6 478,1	5 999,1	244,4	252,6	0,023
315	3 509,9	3 401,8	2 669,4	3 686,7	3 509,3	3 401,1	2 668,8	3 686,2	0,022

4.3. *Know limitations*

A limitation of the approximate modular solution is that is not adequate in cases in which the mechanism is near to self-brake, In such configurations the influence of friction forces is important and an approximated estimation comes to big calculation errors. Ref. [25] proposes with respect to this situation: In particular, analyzing a self-break mechanism, by this method is possible to conclude that it do not belongs to this class of mechanisms.

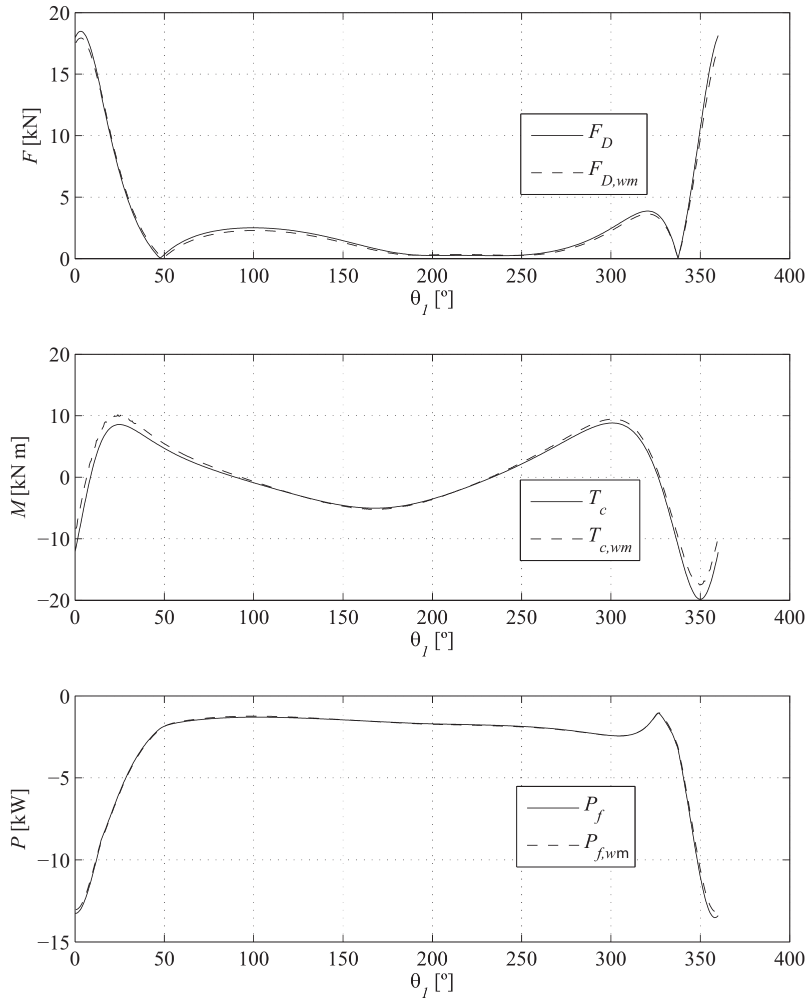

Fig. 15. Four-bar mechanism: Friction power consume (P_f), input required moment (T_c) and D pair reaction (F_D).

5. Conclusion

The performance of a mechanism during its periodic movement depends of the work needed to overcome the productive and non-productive resistances. Usually non productive resistances appears principally because of friction. In mechanisms the calculation of reactions with friction usually comes into a non-linear problem. Therefore approximated methods of calculation as the presented in section 3 are practical and useful.

The analytical solutions for kinematical pair forces with friction effects for structural groups presented in this paper correspond to the so called general purpose programs based on kinematic units. The form of the modules allows to codify computer functions with emphasis placed on the users finding the input data structure easy to understand. Therefore very complex planar linkages may be described using a simple and familiar form of the input data.

For the development of the analytical solutions including friction effects a methodology was proposed. Such a methodology is based in an iterative but practical solution and a parametric CAD graphical validation.

The modules for mechanisms analysis developed by this method preserve the characteristics of the general purpose programs based on kinematic units: computational efficiency and flexibility.

The proposed analytical iterative method is not adequate to analyze mechanisms that are near to self-brake. In that case the influence of friction forces is important and big calculation errors can come.

The modules for the structural groups can be codified in a program like SIMULINK in which each Assur group is represented by an independent block. In this case the analysis of mechanisms comes intuitive and easy because it is possible to follow graphically the structural law of the mechanism. To analyze a particular mechanism the blocks corresponding to the structural groups are dragging and dropping. The law of formation is established wiring the blocks properly.

Future work can be done analysing superior-class structural groups and using more elaborated friction models for the kinematical pairs.

References

[1] H. Ding, Z. Huang, A new theory for the topological structure analysis of kinematic chains and its applications. Mech. Mach. Theory 42 (10) (2007) 1264-1279.

[2] J. Buśkiewicz, A method of optimization of solving a kinematic problem with the use of structural analysis algorithm (SAM). Mech. Mach. Theory 41 (7) (2006) 823-837.

[3] S. Durango, L. J. Mesa, Solución analítica de mecanismos usando grupos de Assur. Scientia et Technica 27 (2005) 121-126

[4] T. S. Mruthyunjaya, Kinematic structure of mechanisms revisited. Mech. Mach. Theory 38 (4) (2003) 279-320.

[5] S. Mitsi, K. -D. Bouzakis, G. Mansour, Position analysis in polynomial form of planar mechanism with an Assur group of class 4 including one prismatic joint. Mech. Mach. Theory 39 (3) (2004) 237-245.

[6] S. Mitsi, K. -D. Bouzakis, G. Mansour, I. Popescu, Position analysis in polynomial form of planar mechanisms with Assur groups of class 3 including revolute and prismatic joints. Mech. Mach. Theory 38 (12) (2003) 1325-1344.

[7] X. Kong, C. M. Gosselin, Forward displacement analysis of third-class analytic 3-RPR planar parallel manipulators. Mech. Mach. Theory 36 (9) (2001) 1009-1018.

[8] L. Han, Q. Liao, C. Liang. Closed-form displacement analysis for a nine-link Barranov truss or a eight-link Assur group. Mech. Mach. Theory 35 (3) (2000) 379-390.

[9] S. Mitsi, Position analysis in polynomial form of planar mechanisms with a closed chain of the Assur group of class 4. Mech. Mach. Theory 34 (8) (1999) 1195-1209.

[10] C. Jinkui, C. Weiqing, Systemics of Assur groups with multiple joints. Mech. Mach. Theory 33 (8) (1998) 1127-1133.

[11] M. R. Hansen, A general method for analysis of planar mechanisms using a modular approach. Mech. Mach. Theory 31 (8) (1996) 1155-1166.

[12] T. Młynarski, Position analysis of planar linkages using the method of modification of kinematic units. Mech. Mach. Theory 31 (6) (1996) 831-838.

[13] M. R. Smith and Z. Ye, Simplified data structure for analysing mechanisms using character handling techniques. Computer-Aided Design 16 (4) (1984) 197-202

[14] G. L. Kinzel and C. Chang, Analysis of planar linkages using a modular approach. Mech. Mach. Theory 19 (1) (1984) 165-172.

[15] V. Bràt and P. Lederer, KIDYAN: Computer-aided kinematic and dynamic analysis of planar mechanisms. Mech. Mach. Theory 8 (4) (1973) 457-467.

[16] L. V. Assur, Investigation of plane hinged mechanisms with lower pairs from the point of view of their structure and classification (in Russian): Part I, II, Bull. Petrograd Polytech. Inst. 20 (1913) 329-386; Bull. Petrograd Polytech. Inst. 21 (1914) 187-283 (vols. 21-23, 1914-1916).

[17] S. Durango, Dynamics of second-class planar mechanisms by means of a computer aided modular approach. Proceedings 23rd ISPE International CARS and FOF. V 1 Boǵota (2007) 421-428.

[18] G. Calle, H. F. Quintero, A. Díaz, Propuesta de enseñanza de la cinemática de mecanismos planos con la ayuda de programas CAD paramétricos. Proceedings Innovación docente CREA'02. La Habana (2002).

[19] G. Calle, H. F. Quintero, A. Díaz, Análisis cinemático de mecanismos planos a partir del análisis estructural según Assur, Proceedings V Congreso Iberoamericano de Ingeniería Mecánica. Mérida (2001).

[20] S. Molian, Proc. Mechanisms Conf. V 2 Granfield (1984).

[21] D. B. Marghitu, M. J. Crocker. Analytical elements of mechanisms, Cambridge University Press, Cambridge 2001.

[22] R. L. Norton, Diseño de Maquinaria, 2 ed, McGraw Hill, México, 1995.

[23] P. E. Nikravesh, Computer aided analysis of mechanical systems, Prentice Hall, New Jersey, 1988.

[24] I. I. Artobolevski, Theory of mechanisms and machines, Nauka, Moscu, 1988. (In Russian)

[25] G. G. Baranov, Curso de la teoría de mecanismos y máquinas, 2 ed. MIR, Moscú, 1979, pp. 403-404.