BI-CURVE AND MULTI-PATCH SMOOTHING WITH APPLICATION TO THE SHIPYARD INDUSTRY

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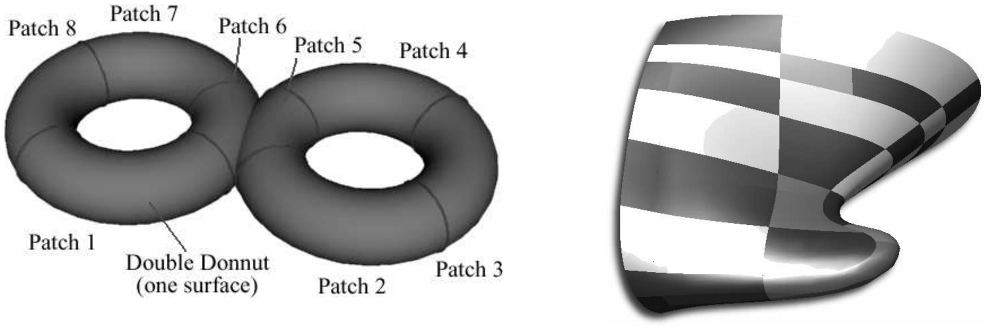
ABSTRACT

Algorithms are proposed and implemented in a commercial system which allow for the C^1-continuity matching between adjacent B-spline curves and B-spline patches. These algorithms only manipulate the positions of the control points, therefore respecting the constraint imposed by the sizes of the available commercial steel plates. The application of the algorithms respect the initial hull partition made by the designers and therefore the number and overall shape and position of the constitutive patches remains unchanged. Algorithms were designed and tested for smoothing the union of (a) two B-spline curves sharing a common vertex, (b) two B-spline surfaces sharing a common border, and (c) four B-spline surfaces sharing a common vertex. For this last case, an iterative heuristic degree-of-freedom elimination algorithm was implemented. Very satisfactory results were obtained with the application of the presented algorithms in shipyards in Spain.

1 Introduction and Literature Review

B-spline curves and surfaces have been used extensively in the past to define ship-hull geometry for design purposes [1, 2]. The popularity of B-spline for free-form surface design lies in their useful characteristics, such as local support, the convex hull, and variation-diminishing properties [3]. Theoretical background of B-spline curves and surfaces can be found in Farin [4]. A discussion of their suitability for ship hull surface definition can be found in Rogers [5].

Applications of e.g. Computational Fluid Dynamics use single patch representations, which solve the issue of smoothness by itself [6], but do not reflect that the manufacture and assembly are performed with smaller standard plates, as produced in the steel mills. Also, fitting the complex surface of a ship hull with a single B-spline patch may lead to either an inaccurate representation, or a designer-unfriendly representation i.e. a single patch with a high number of control points. On the other hand, since a single B-spline patch can only represent surfaces of simple topological type, a surface of arbitrary topological type (see Figures 1(a) and 1(b)) must be defined as a set of B-spline patches [7]. The set of patches must constitute a partition of the ship hull surface and must also maintain tangent plane continuity (C^1 continuity) across neighboring patches. Enforcing C^1 continuity between adjacent patches while at the same time fitting the patch network to the points (of the ship hull surface in this case) is a challenging problem [7].



(a) Partition of a 2-genus 2-Manifold (double donnut)

(b) Partition of Ship Bow

Figure 1: Non-rectangular Partition of 2-manifolds with Rectangular Patches

Loop [8] presents an algorithm for creating a smooth set of rectangular and triangular spline surfaces, starting with an irregular mesh of polygonal flat faces. The algorithm takes into consideration curvature parameters to decide the tiling or merging of patches. The final result may have spline patches of sizes and shapes dictated by the curvature criteria. Because of this characteristic, the algorithm is not suitable to be applied in the problem at hand, in which one must respect the constraint posed by the predefined plates with which the hull is to be constructed.

Ball [9] and Peters [10] derive continuity conditions for the subdivision of surfaces. Ball uses Fourier transform-based techniques to do so. Peters presents a method for verifying smoothness of subdivided B-spline surfaces generated using Doo-Sabin [11] and Catmull-Clark [12] subdivision algorithms. In our case, subdivision is not only unnecessary but also not allowed, since the steel plates to manufacture the hull are pre-defined. Our goal is to respect the collection of B-spline patches, and to slightly modify their control points to achieve C^1 continuity among them.

Bardis [3] presents an algorithm for C^1 continuity between adjacent patches which requires the merging of all the knot vectors of the B-spline patches, the unification of the order and of the number of vertices of the control polygons, and the use of arbitrarily selected scalar functions called *bias*. Hence, it was not compliant with our goal of smoothing B-splines by modifying only their control points.

For the making of software for the shipbuilding industry no explicit algorithms for B-spline curve and surface smoothing were found in the reviewed literature. It thus became necessary to design and implement own algorithms for this task. It is the purpose of this paper to present the designed algorithms for B-spline curve and surface smoothing, together with the results obtained to smooth real ship B-spline surface patches. The paper is structured as follows: Section 2 presents a brief description of the ship hull surface modeling process using B-spline curves and surfaces. Section 3 presents an algorithm for B-spline curves smoothing. Section 4 presents two algorithms for B-spline surfaces smoothing: one for two adjacent surfaces sharing a common border, and one for four surfaces incident to a common vertex. Conclusions are presented in section 6.

2 Hull surface modeling using a set of B-spline surfaces

The computer modeling of a ship hull is performed, in our case, from the ship hull lines. These lines are planar curves in $\mathbb{R}^3$ resulting from the intersection of the ship hull surface against cross sections perpendicular to the axes of the ship coordinate system. The modeling process is roughly as follows: (i) A set of B-spline curves is manually fitted to ship hull lines. Several rectangular regions on the ship hull surface result from this process, as shown in figure 1. (ii) Rectangular B-spline patches are generated from the four B-spline curves that enclose each of these regions. An initial model of the ship hull surface, constituted by a network of C^0-continuous rectangular B-spline patches is thus obtained. (iii) Each pair of adjacent patches is smoothed using the implementation of the algorithm described in section 4.2.1. Every set of four patches sharing a common vertex is also smoothed using the implementation of the algorithm described in section 4.2.2. The final result of the process is a set of rectangular B-spline patches whose union is C^1-continuous, and constitutes the final model of the ship hull surface (see Figure 8).

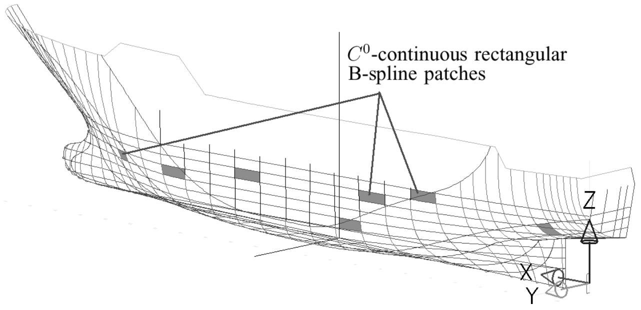


Figure 2: Set of B-spline curves interpolating the ship lines and local C^0 B-Spline patches

3 Methodology. Smoothing of B-spline curves in shared vertices

3.1 Condition for C^1 continuity between B-Spline curves

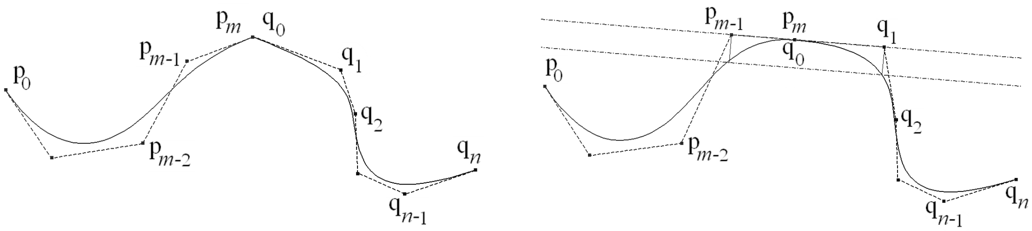
Let P and Q be two B-Spline curves in $\mathbb{R}^3$. Let $S_P = \{\mathbf{p}_0, \mathbf{p}_1, ..., \mathbf{p}_m\}$ and $S_Q = \{\mathbf{q}_0, \mathbf{q}_1, ..., \mathbf{q}_n\}$, $\mathbf{p}_i, \mathbf{q}_i \in \mathbb{R}^3$, be the sequences of control points of P and Q, respectively. If $\mathbf{p}_m$=$\mathbf{q}_0$, i.e. P and Q are C^0-continuous at $\mathbf{p}_m$, then P and Q are also C^1-continuous at $\mathbf{p}_m$ if $\mathbf{p}_{m\text{-}1}$, $\mathbf{p}_m$, and $\mathbf{q}_1$ are collinear, and $\mathbf{p}_m$ lies between $\mathbf{p}_{m\text{-}1}$ and $\mathbf{q}_1$, i.e. if there exists $\lambda \in (0,1) \subset \mathbb{R}$ such that

$$\mathbf{p}_m = \mathbf{q}_0 = (1\text{-}\lambda)\mathbf{p}_{\text{m-1}} + \lambda\mathbf{q}_1 \qquad (1)$$

3.2 Algorithm for C^1 continuity between curves

Given two separate B-Spline curves P and Q in $\mathbb{R}^3$ connected at a common endpoint $\mathbf{p}_m$=$\mathbf{q}_0$ (see figure 2), the **goal** of a curve smoothing process is to determine new positions for the control points of P and Q so that the two curves become C^1-continuous at $\mathbf{p}_m$, i.e. the normalized direction vectors of P and Q at $\mathbf{p}_m$ are equal. If the union of the curves P and Q is required to be smoothed at point $\mathbf{p}_m$, and $\mathbf{p}_{m\text{-}1}$, $\mathbf{p}_m$ and $\mathbf{q}_1$ are not collinear, at least one of these three points must be moved in order to do so. Although infinite solutions to this problem exist (there are infinite ways of arranging three points to lie in a same line), some of them are more suitable for design and construction purposes. For instance, sometimes the shared control point is desired to remain fixed (see figure 3(b)).

Suppose that we want to force $\mathbf{p}_{m\text{-}1}$, $\mathbf{p}_m$, and $\mathbf{q}_1$ to lie in the same line, by moving $\mathbf{p}_{m\text{-}1}$ and $\mathbf{q}_1$ to new positions $\mathbf{p}^*_{m-1}$ and $\mathbf{q}^*_1$, and leaving $\mathbf{p}_m$ fixed. A way to calculate $\mathbf{p}^*_{m\text{-}1}$ and $\mathbf{q}^*_1$ is as follows: Let L be the line passing through $\mathbf{p}_{m\text{-}1}$ and $\mathbf{q}_1$, and L^* be the line passing through $\mathbf{p}_m$ and parallel to L. Let $\Pi_{\mathbf{p}_{m-1}}$ and $\Pi_{\mathbf{q}_1}$ be the planes with normal vector $\hat{\mathbf{n}}$ and respective pivot points $\mathbf{p}_{m\text{-}1}$ and $\mathbf{q}_1$, where $\hat{\mathbf{n}} = (\mathbf{q}_1 - \mathbf{p}_{m-1})/(|\mathbf{q}_1 - \mathbf{p}_{m-1}|)$. It can be seen that possible values for $\mathbf{p}^*_{m\text{-}1}$ and $\mathbf{q}^*_1$ that satisfy equation 1 are given by $\mathbf{p}^*_{m-1} = \Pi_{\mathbf{p}_{m-1}} \cap L^*$ and $\mathbf{q}^*_1 = \Pi_{\mathbf{q}_1} \cap L^*$.



(a) B-Spline with C^0 continuity at $\mathbf{p}_m$=$\mathbf{q}_0$

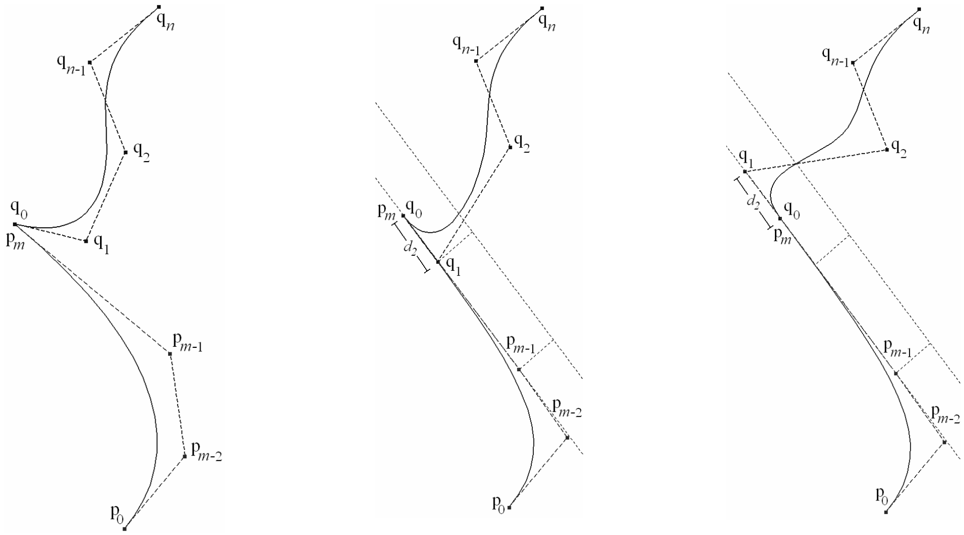
(b) B-Spline with C^1 continuity at $\mathbf{p}_m$=$\mathbf{q}_0$

Figure 3: C^1 Continuity between adjacent B-Spline curves by adjusting $p_{m\text{-}1}$ and q_1

3.2.1 Exception Treatment

Let λ^* be the value of λ at which $\overset{*}{\mathbf{p}}_{m\text{-}1}$ and $\overset{*}{\mathbf{q}}_1$ satisfy equation 1. Because the procedure described above does not ensure that $\lambda^* \in (0,1)$, an additional step becomes necessary. If $\lambda^* \notin (0,1)$, then $\mathbf{p}_m$ does not lie between $\overset{*}{\mathbf{p}}_{m\text{-}1}$ and $\overset{*}{\mathbf{q}}_1$. It is necessary to force $\mathbf{p}_m$ to lie between $\overset{*}{\mathbf{p}}_{m\text{-}1}$ and $\overset{*}{\mathbf{q}}_1$. Since $\mathbf{p}_m$ is required to remain fixed, $\overset{*}{\mathbf{p}}_{m\text{-}1}$ or $\overset{*}{\mathbf{q}}_1$ should be moved again. To avoid an excessive change in the geometry of the curves, the point to be moved will be the one that lies the closest to $\mathbf{p}_m$.

Let $d_1 = \left|\mathbf{p}_m - \overset{*}{\mathbf{p}}_{m-1}\right|$ and $d_2 = \left|\mathbf{p}_{\mathrm{m}} \text{-} \overset{*}{\mathbf{q}}_1\right|$. If $d_1 \leq d_2$, $\overset{*}{\mathbf{p}}_{m\text{-}1}$ will be moved to a final position $\overset{**}{\mathbf{p}}_{m-1} = \mathbf{p}_m + \left(\mathbf{p}_m - \overset{*}{\mathbf{p}}_{m-1}\right)$. If $d_1 > d_2$, $\overset{*}{\mathbf{q}}_1$ will be moved to a final position $\overset{**}{\mathbf{q}}_1 = \mathbf{p}_m + \left(\mathbf{p}_{\mathrm{m}} \text{-} \overset{*}{\mathbf{q}}_1\right)$ (see Figures 4(a)-4(c)).



(a) B-Spline curves with C^0 continuity at $\mathbf{p}_m = \mathbf{q}_0$

(b) B-Spline curves after algorithm without correction

(a) Curves become C^1 continuous after correction

Figure 4: Exception Treatment. Continuity between adjacent B-Spline Curves

4 Methodology. Smoothing of B-Spline Surfaces in shared borders

4.1 Condition fo C^1 continuity between B-Spline surfaces

Let A be a B-Spline surfaces and P^A the array of control points of A,

$$P^A = \begin{bmatrix} \mathbf{p}_{11}^A & \mathbf{p}_{12}^A & \cdots & \mathbf{p}_{1n}^A \\ \mathbf{p}_{21}^A & \mathbf{p}_{22}^A & \cdots & \mathbf{p}_{2n}^A \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{p}_{m1}^A & \mathbf{p}_{m2}^A & \cdots & \mathbf{p}_{mn}^A \end{bmatrix}$$

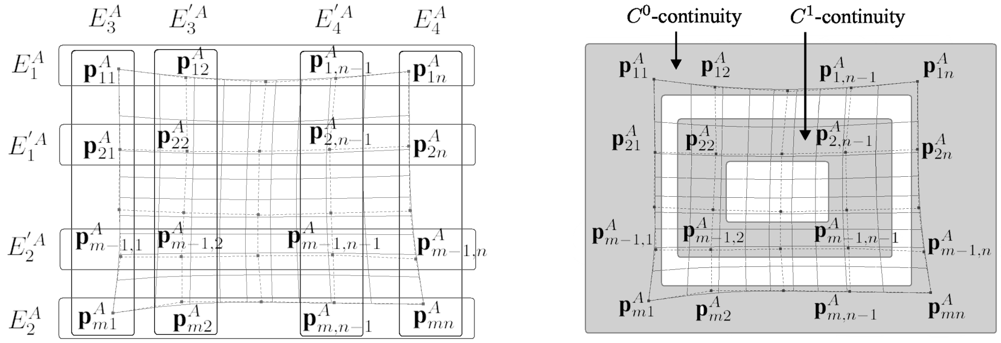
where $\mathbf{p}_{ij}^A \in \square^3$.

Definition. *Alignment* of PL curves.

Let $E_1=[\mathbf{p}_{11},\mathbf{p}_{12},...,\mathbf{p}_{1n}]$, $E_2=[\mathbf{p}_{21},\mathbf{p}_{22},...,\mathbf{p}_{2n}]$ and $E_3=[\mathbf{p}_{31},\mathbf{p}_{32},...,\mathbf{p}_{3n}]$ be three sequences of control points, where $\mathbf{p}_{ij}\in R^3$. We say that E_1, E_2 and E_3 are *aligned* if for all j=1,2,...,n, the points $\mathbf{p}_{1j}$, $\mathbf{p}_{2j}$ and $\mathbf{p}_{3j}$ are collinear exactly in that order, i.e. satisfy equation $\mathbf{p}_{2j}=(1\text{-}\lambda)\mathbf{p}_{1\mathrm{j}}+\lambda\mathbf{p}_{3\mathrm{j}}$ with $\lambda\in(0,1)$.

The boundary control point sequences for A are $E_1^A=\left[\mathbf{p}_{11}^A,\mathbf{p}_{12}^A,...,\mathbf{p}_{1n}^A\right]$, $E_2^A=\left[\mathbf{p}_{m1}^A,\mathbf{p}_{m2}^A,...,\mathbf{p}_{mn}^A\right]$, $E_3^A=\left[\mathbf{p}_{11}^A,\mathbf{p}_{21}^A,...,\mathbf{p}_{m1}^A\right]$ and $E_4^A=\left[\mathbf{p}_{1n}^A,\mathbf{p}_{2n}^A,...,\mathbf{p}_{mn}^A\right]$. Let B be another B-Spline surface. We say that the control points of the i-th border of A are equal to the control points of the j-th border of B, if there exist $i,j\in\{1,2,3,4\}$, such that $E_i^A=E_j^B$ or $E_i^A=E_j^{*B}$, where E_j^{*B} is the reverse-order vesion of E_j^B. A necessary but not sufficient condition for A to be C^0-continuous with B at the i-th border of A and the j-th border of B is that the control points of these two borders be equal.

Let us also define a sequence of control points $E_i^{'A}$ associated to each border E_i^A, for i=1,2,3,4, as per figure 5. $E_1^{'A}=\left[\mathbf{p}_{21}^A,\mathbf{p}_{22}^A,...,\mathbf{p}_{2n}^A\right]$, $E_2^{'A}=\left[\mathbf{p}_{m-1,1}^A,\mathbf{p}_{m-1,2}^A,...,\mathbf{p}_{m-1,n}^A\right]$, $E_3^{'A}=\left[\mathbf{p}_{12}^A,\mathbf{p}_{22}^A,...,\mathbf{p}_{m2}^A\right]$ and $E_4^{'A}=\left[\mathbf{p}_{1,n-1}^A,\mathbf{p}_{2,n-1}^A,...,\mathbf{p}_{m,n-1}^A\right]$.



(a) Sequences of control points E$_{0A\ i}$ associated to each E$_{Ai}$, for i = 1, 2, 3, 4

(b) Control points governing C$_0$ and C$_1$ continuity

Figure 5: Sequences of control points in A

Let A be C^0-continuous with B, at the i-th border of A and the j-th border of B. This implies that $E_i^A=E_j^B$ or $E_i^A=E_j^{*B}$. Unless otherwise stated, two surfaces "being C^0-continuous" means that they meet at border i (in A) and j (in B). Also we assume WLOG that $E_i^A=E_j^B$ (the vertices are enumerated in identical order). The same observation holds for C^1 continuity. We say that A is C^1-continuous with B, if (i) $E_i^{'A}$, E_i^A, and $E_j^{'B}$ are aligned exactly in that order, and (ii) the rows of aligned control points are parallel to each other.

4.2 Algorithms for surface C^1 continuity

Two different smoothing processes are identified here. The basic surface-smoothing process consists in achieving C^1 continuity between two surfaces at their common border, i.e. the border at which the surfaces are C^0-continuous. A second process consists in achieving C^1 continuity between four pairwise-C^0-continuous surfaces sharing a vertex, at their common borders.

4.2.1 C^1 continuity between two surfaces at a common border

Given two separate B-Spline surfaces A and B in $\square^{3}$, connected at a common *border*, $E_i^A = E_j^B$, the **goal** of a surface-smoothing process is to determine new positions for the control points of A and B so that the two surfaces become C^1-continuous at their common border. The procedure is to make parallel the rows of control points that are to be aligned, and then to make collinear the $E_{ik}^{'A}$, E_{ik}^{A}, $E_{jk}^{'B}$ points for k=1...m, that is, to pairwise align the control points at the seam between the two patches (m is the number of control points of such borders).

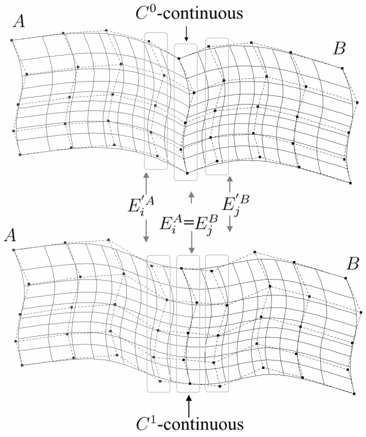


Figure 6: $C^{0,1}$ -continuity between A and B at i-th border of A, and j-th border of B

4.2.2 C^1 continuity between four surfaces at common vertex

Let A, B, C, and D be adjacent B-Spline surfaces, meeting at one vertex. The meeting borders among them are: $E_i^A = E_k^B$, $E_l^B = E_m^C$, $E_n^C = E_o^D$, $E_p^D = E_j^A$. The common vertex is $P_{i^* j^*}^{A} = P_{k^* l^*}^{B} = P_{m^* n^*}^{C} = P_{o^* p^*}^{D}$ Subscripts take values between 1 and 4.

The arrangement of surfaces A, B, C, D, shown in figure 7 satisfies the previous conditions, since the four surfaces are pairwise-C^0-continuous and have a common control point that belongs to all the borders at which the surfaces are C^0-continuous.

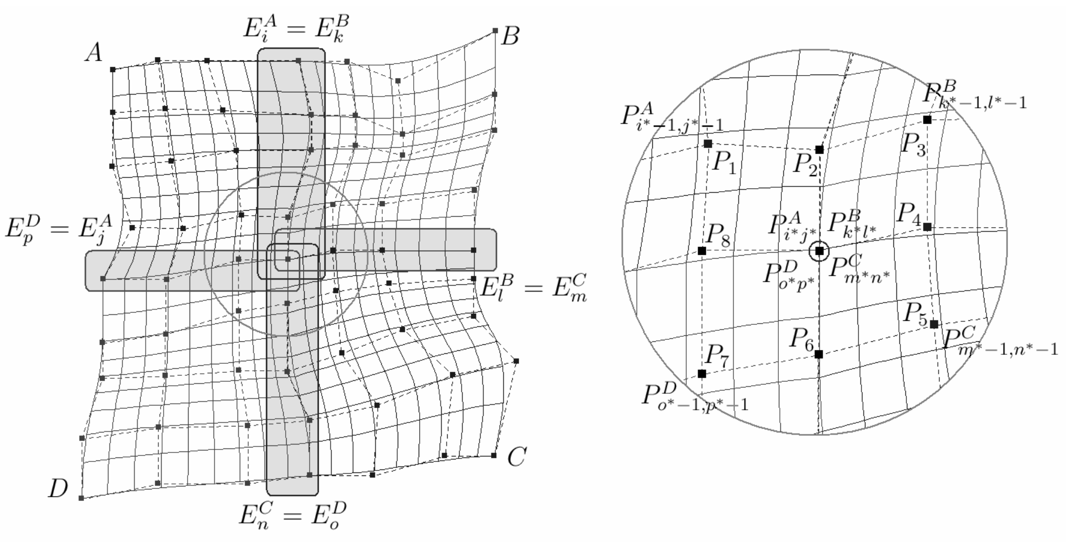


Figure 7: C^0-continuous set of four adjacent B-Spline surfaces meeting at a common point

Given four B-Spline surfaces A, B, C, and D in $\mathbb{R}^3$, satisfying conditions mentioned above, the **goal** of a surface-smoothing process is to determine new positions for the control points of A, B, C and D, so that the union of the four surfaces becomes C^1-continuous.

Separately achieving pairwise-C^1 continuity between the four B-Spline surfaces includes calculating correct modified positions of the controls points of A, B, C and D. However, such a process does not correctly calculate the positions for the common point (P_0) and its surrounding 8 vertices ($P_1,...,P_8$ in Figure 7).

Algorithm 1 calculates the modified positions of $P_0, P_1,..., P_8$ such that C^1 Continuity among the union of A, B, C and D is achieved. This algorithm is based on the fact that if $P_0, P_1,..., P_8$ lie on the same plane, and the elements in each of the following sequences $s_1 = [P_1, P_2, P_3]$, $s_2 = [P_3, P_4, P_5]$, $s_3 = [P_5, P_6, P_7]$, $s_4 = [P_7, P_8, P_1]$, are collinear exactly in that order, then C^1 Continuity is achieved at points $P_0, P_1,..., P_8$. For the sake of compactness in the article we omit the proof of convergence for algorithm 1.

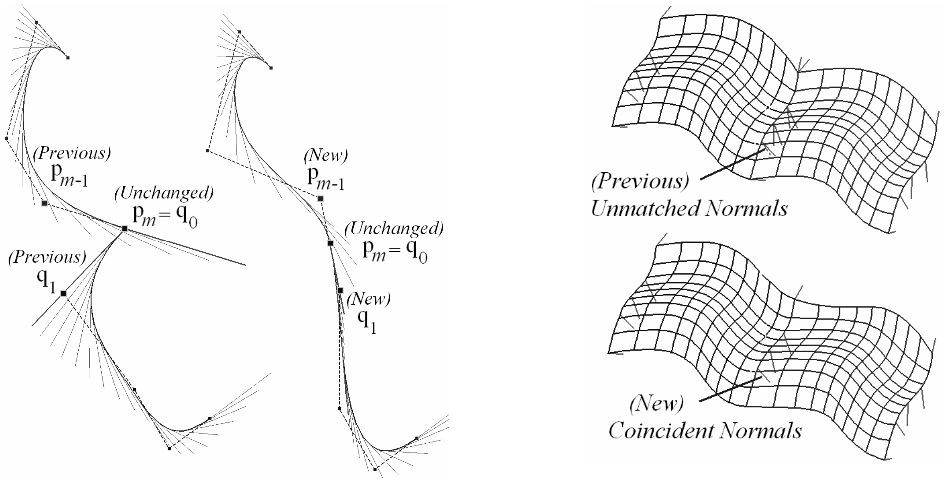
Algorithm 1 C^1 continuity between four surfaces

```
Identify values of $i, j, k, l, m, n, o, p$
Pairwise-smooth surfaces $A$ with $B$, $B$ with $C$, $C$ with $D$, $D$ with $A$
Calculate best-fit plane $\Pi^*$ for points $P_0, P_1, \ldots, P_8$
Project points $P_0, P_1, \ldots, P_8$ into $\Pi^*$
while $P_1, P_3, P_5, P_7$ do not converge do
    Move $P_1$ to make $P_1, P_2, P_3$ collinear (algorithm in section 3.2)
    Move $P_3$ to make $P_3, P_4, P_5$ collinear
    Move $P_5$ to make $P_5, P_6, P_7$ collinear
    Move $P_7$ to make $P_7, P_8, P_1$ collinear
end while
```

5 Results

A large number of adjacent B-spline curves were smoothed using the industrial implementation of the algorithm described in section 3. After the algorithm was applied, the upper bound of the angular deviation between tangent vectors at the boundary of matched curves was 2.9×10^{-5} degrees (figure 8(a)).

Likewise, a large number of adjacent B-spline surfaces were smoothed using the algorithm described in section 4.2.1. The relative error between the normal vectors of both surfaces along their common border remained below 10^{-5} degrees (figure 8(b)). Figure 9 shows the final result of the 4-patch smoothing algorithm.



(a) Vectors tangent to two adjacent B-spline curves, before and after being smoothed

(b) Vectors Normal to two adjacent surfaces, before and after being smoothed

Figure 8: Tangent and Normal vectors to B-splines used for C^1 continuity testing

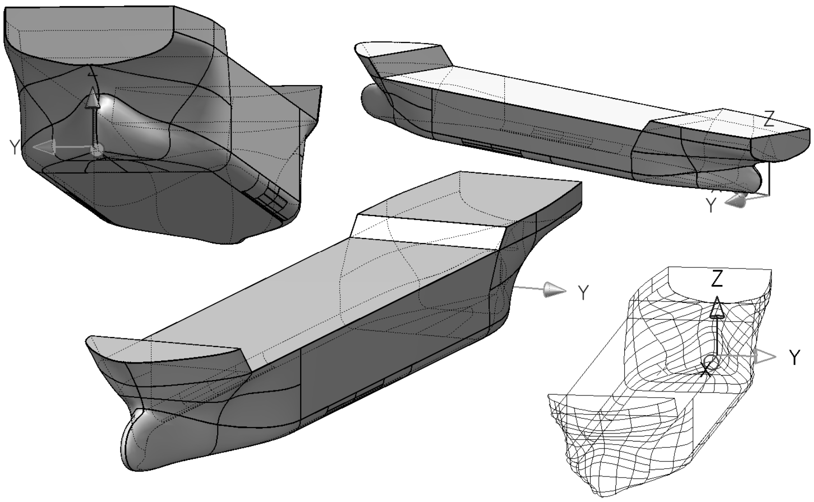


Figure 9: Ship hull surface obtained through the procedure described in section 2

6 Conclusions

Industrially implemented algorithms for B-spline curve and surface smoothing were discussed in this paper. The algorithms achieve C^1 continuity between adjacent curves and surfaces by modifying only the positions of their control points. The main advantages of the presented algorithms are their simplicity, which results in their easy implementation and modification, and the fact that properties of the curves and surfaces such as their order and their poles remain unchanged. Several tests were made to the obtained smoothed curves and surfaces, based on the tangent and normal vectors of the B-spline at their common point or border. The relative error between the components of the tangent and normal vectors was in all test cases below 10^{-5} degrees. Several real ship hull surfaces have been modeled at the Design and Engineering Group (GED), Universidade de Vigo, following the discussed methodology. One of these models was presented in this paper.

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