BOOLEAN 2D SHAPE SIMILARITY FOR SURFACE RECONSTRUCTION[1]

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Abstract

Surface reconstruction problem (SRP) from planar samples has been traditionally approached by either (i) using local proximity between data points in adjacent layers, or by (ii) classifying the topological transitions that may explain the evolution of the cross sections. Strategy (i) is robust in the sense that it has answers for every possible case, although in some scenarios renders counterintuitive surfaces, commented below. Approach (ii) has mainly remained in the theoretical terrain. The present work follows on aspect (ii), by using a Morse-based topological classification of the transitions, and complementing it with reasoning based on the geometry of the evolving cross sections to determine a high level description of the transitions from ***m*** to ***n*** contours (***m:n*** transitions). This reasoning of shape similarity is performed by boolean operators. Finally, the surface is synthesized using the ***m:n*** transitions. This conjunction of topological and geometrical reasoning renders highly intuitive results, and allows for the incorporation of methods derived from the area of machine vision.

1. Introduction

Shape reconstruction from planar samples is important because these data are characteristic from medical imaging (MRI, CAT), laser samples, and CMMs (Coordinate Measuring Machines), among others. The problem can be stated as follows: given a set of parallel

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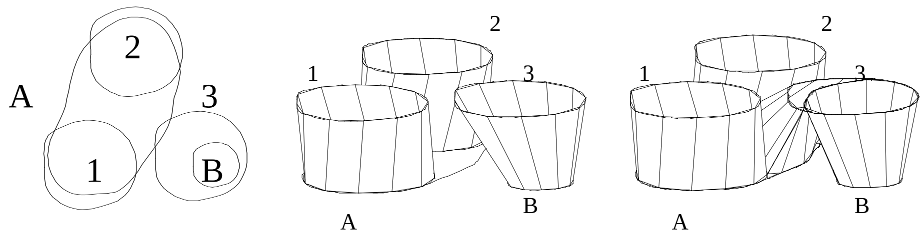
cross sections of an object, expressed as a partition of the original sample into coplanar (or nearly coplanar) sets of unordered points, produce a topologically and geometrically consistent 2-manifold, possibly with boundary (i.e. a partial surface). In the present scope, C_0 continuity is sufficient, leaving for downstream applications the issue of C_1 or C_2 smoothing of the manifold. The problem therefore encompasses the recovery of the planar cross sections into a hierarchical structure (a forest) that accommodates disjoint and nested filled regions (trees). The mapping of regions in level i to regions in level $i+1$ follows. In many methods, the mapping itself realizes the surface, while in others (like the one presented here), the problem of region mapping is divorced from the one of surface construction. In this work, the relations among contours of the two levels are first determined by applying Morse theory and shape similarity, and then, a skin from m to n contours is materialized. The method presented allows the application of tools already developed for computer vision, and also adheres more to geometrical intuition than local - proximity - based methods.

In this article section 2 presents a literature review, while section 3 discusses the main aspects of the methodology used. Section 4 documents the results obtained and concludes the article.

2. Literature Review

Because of space limitations, this section will not discuss methods for contour recovery from unordered sets of points from planar samples (see for example [Amenta et al.98]). However, this problem is relevant if neighboring sample points on the object contour appear as unrelated in the data stream (CMMs and range images). The result of contour reconstruction is a set of trees (forest), where each tree represents a connected bounded



(a) (b) (c)

Fig. 1 - (a) Original contours {A,B} (lower level) must be matched against {1,2,3} (upper level). (b) Intuitive match. For simplicity the tiling in polygon A is not shown. (c) Barequet-Sharir match.

region of the plane, with branching representing the recursive inclusion of regions within regions. External boundaries are traversed in contrary direction as internal ones ([Barequet et al.96, Ruiz.00a, Boissonat.88]).

In general terms, a set $S=\{p_1, p_2,.., p_N\}$ of points is sampled from a 2-manifold (possibly with boundary) embedded in E^3. The sampling interval in every direction must be smaller than half of the smallest characteristic dimension in such direction that one is interested to preserve and model ([Shannon.49]).

2.1 Volume Based Methods

Assume that a scalar field f is available ($f(x,y,z) \in R, (x,y,z) \in [x_0, x_f] \times [y_0, y_f] \times [z_0, z_f]$), where f represents a property like density, absorption, etc., that differentiates an object from its surroundings and within itself. The surface to be reconstructed is in this case an iso-surface $f(x,y,z) = c$. In this case, the Marching Cubes algorithm ([Lorensen et al.87]) seems to be the most popular to build a piecewise-linear surface.

2.2 Reconstruction Methods for Planar Samples

Typical examples of patterned samples come from: (a) CMM (Coordinate Measurement Machines) with a locked degree of freedom; (b) medical imaging; (c) range pictures. The obvious advantage in processing patterned data is the exploitation of neighborhood information implicit in it. In (a) and (b) the data has a planar pattern; in (c) the pattern is a rectangular grid ([Neugebauer.97, Curless et al.96, Ruiz et al.0b]). This paper concentrates on planar data.

Given two contiguous planar, (quasi) parallel cross sections of the object on levels ***i*** and ***i+1***, with sets ***Li*** and ***Li+1*** respectively of embedded 2D contours, a mapping between contours is required, expressing the object surface. A planar sample of this surface should render the original contours one started with. The contours may be nested (recursively contained) and are usually coded as internal / external by the CW / CCW direction of their traversal.

2.2.1 Point proximity algorithms in local neighborhoods of the contours. The most popular methods are based on Voronoy diagrams and Delone triangulations. Some are based on joining regions (points or edge paths) within the contours in one level with their closest counterparts in the opposite level. [Boisonnat et al.93] use an augmented set of points to find 2D Delone triangulations of the filled regions in both levels. Then, a proximity heuristic is used to build tetrahedra from vertices, points or edges contributed from both sides. [Barequet et al.96]) apply spatial closeness and traversal direction (CCW / CW) to sequences of contour edges in level ***i*** that can be matched with sequences on level ***i+1***. The matching path fragments are joined with triangles. The remaining paths are threaded into non-planar loops, called *clefts*, which are tiled with triangles following several optimization criteria. These methods are robust since they consider all situations. However, they construct surfaces based on local proximity, even in cases in which the global situation indicates that such surfaces are counterintuitive (see Fig. 1). None of the commented methods precludes self-intersection of the constructed surface between consecutive slices.

2.2.2 Topological reasoning on contour relations. In contrast with proximity algorithms, topological reasoning implies that the SRP is divided into two stages: (a) which contours from sets *Li* and *Li+1* must be matched (i.e. form the boundary of a connected surface); (b) once a correspondence of *m:n* contours is determined, how to actually calculate the skin that realizes the *m:n* transition. In the surveyed literature, [Shinagawa et al.91a,91b] partially answer question (a) while answers to (b) are suggested. [Shinagawa et al.91a] use Morse theory to build a classification of the possible topological events (Morse indexes and Reeb graphs ([Morse.34, Fomenko et al.97])) that could occur between two consecutive cross sections. As a result, a series of conceptual operators (addition of 0,1 or 2-handles, Fig. 2) are presented, which cause the evolution of the cross sections . The record of these topological events encodes the surface. The code describes not only the evolution of cross sections but also the operations producing changes in the contours. However, the history of Morse indexes and the Reeb Graph do not uniquely define the position and shape of the manifold. These authors propose for future work a homotopy model to reconstruct the surface (aspect (b)). [Shinagawa et al.91b] construct the Reeb Graph from geometrical considerations such as similarity of contours, therefore completing one part of the geometrical aspect of [Shinagawa et al.91a]. However, neither of the two contributions actually constructs the surface.

The main purpose of the present article is to (i) infer the handle additions necessary to pass from level ***i*** to level ***i+1***, (ii) convert the handle information into the determination of pairs of ***m*** contours in level ***i*** and ***n*** in level ***i+1*** that must be surfaced or lofted, (iii) realize the surface by: (a) using 0, 1 or 2- handles and boolean operators, or (b) ***m:n*** contour lofting.

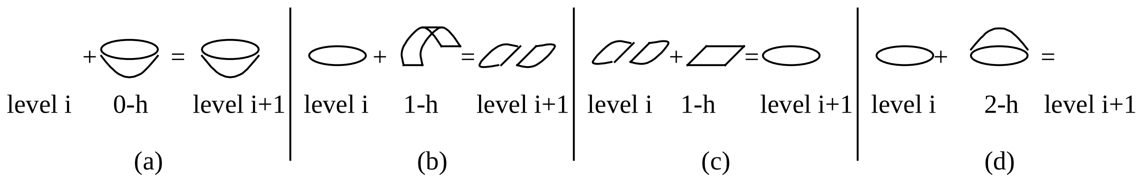


Fig. 2 - Morse transitions and their effects on contour population. (a) Creation: add 0-handle. (b) Splitting: add 1-handle, (c) Merging: add 1-handle, (d) Annihilation: add 2-handle.

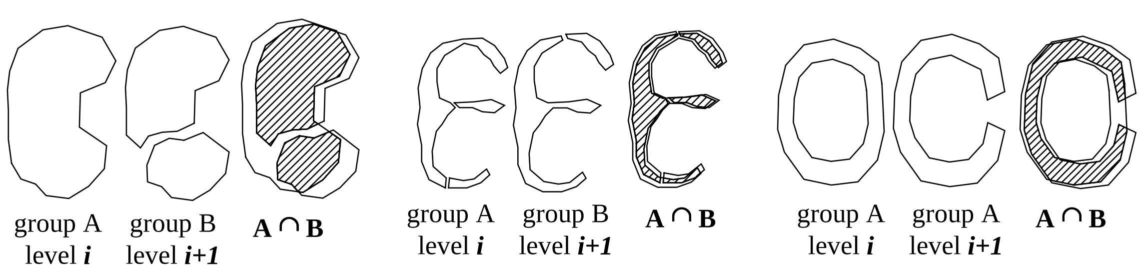
3. Methodology

This paper will attack the SRP by (i) establishing similarity between sets of complete contours of ***Li*** and ***Li+1*** and (ii) constructing the skin among the matched contours by a generalized ***m:n*** loft..

3.1 Cross Section Similarities

In contrast with [Boissonat et al.93] and [Barequet et al.96], the present approach seeks to formulate conjectures of similarity between 2D regions. The goal is to evaluate whether a set of ***m*** contours taken together resembles another set of ***n*** ones.

Several criteria were used to evaluate whether two regions ***A, B*** (each formed by possibly several polygons) are similar, and placed frontally enough to each other (Fig. 3). The two best indicators were: (i) $f_1(A,B) = area(A \cap B) / max(area(A), area(B))$, and (ii) $f_2(A,B) = area(A \cap B) / average(area(A),area(B))$. If the regions are equal and they face each other,



(a) Add one 1-handle. (b) Add three 1-handles. (c) Add one 1-handle.

Fig. 3. - Intersection of group contour areas for similarity calculation and topological operations to produce level ***i+1*** from level ***i***.

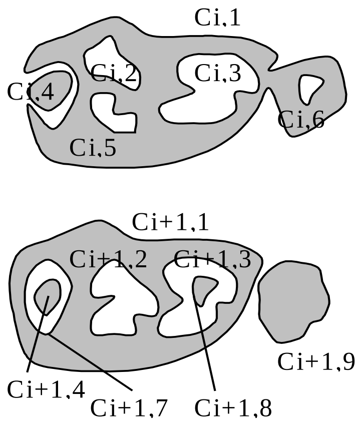


Fig. 4 – Cross sections of levels ***i*** and ***i +1***.

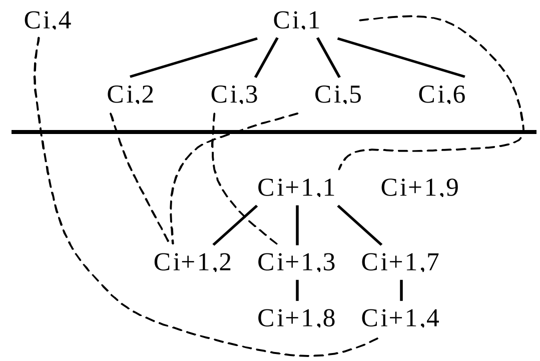
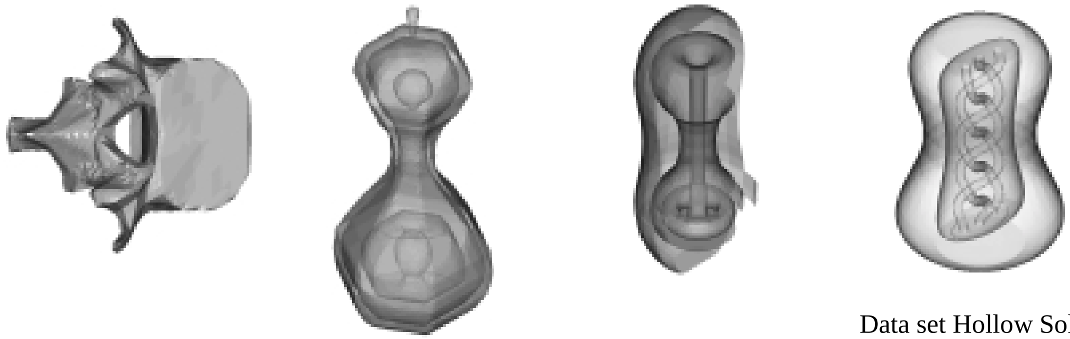


Fig.5 – Inter-level contour mapping between levels ***i*** and ***i+1***.

these factors have value 1. As such condition degrades, they tend to 0. Indicator $\boldsymbol{f_2}$ is currently used and its performance is very satisfactory. From now on $\boldsymbol{f_2}$ will be denoted by $\boldsymbol{f}$. This indicator of similarity is evaluated after deletion of obviously impossible matches (for example, if their mini-max bounds do not intersect). Notice that $\boldsymbol{f(A,B) = f(B,A)}$ and therefore it is independent from the numbering of the levels.

3.2 Forest Mapping

Fig. 4 shows two typical contiguous cross sections, while Fig. 5 presents their forest representation. The goal of the forest mapping algorithm is to establish a relation between these two graphs (Fig. 5). The relation is expressed as a set of pairs $\boldsymbol{(A,B)}$ such that the groups of contours $\boldsymbol{A=\{c_{i,k1} ,...,c_{i,kr} \}}$ from level $\boldsymbol{i}$ and $\boldsymbol{B=\{c_{i+1,q1} ,...,c_{i+1,qs}\}}$ from level $\boldsymbol{i+1}$ satisfy a minimum threshold value on the similarity evaluation ($\boldsymbol{f(A,B) \in [f_{min},1]}$). For example, a result of the algorithm is to determine that $\boldsymbol{A=\{ c_{i,1} , c_{i,4} \}}$ from level $\boldsymbol{i}$ and $\boldsymbol{B=\{c_{i+1,1} , c_{i+1,9} , c_{i+1,4} \}}$ must be matched. The algorithm considers all pairs $\boldsymbol{(A,B)}$, where $\boldsymbol{A \subseteq L_i}$ and $\boldsymbol{B \subseteq L_{i+1}}$. First, a strict value of the similarity $\boldsymbol{f_{min}}$ threshold is required to proclaim $\boldsymbol{(A,B)}$ a match. Then, lower values are allowed. The $\boldsymbol{(A,B)}$ couples are sorted to give priority of consideration to simple matches first, and then to complicated ones. When a



Data set Vertebra. Recovered Model

Data set Pear. Recovered Model

Data set Multi-Torus. Recovered Model

Data set Hollow Solid with inner braid. Recovered Model

Fig. 6 - Results

match is found, the children of the matched groups, ***children(A)***, and ***children(B)*** are recursively considered, in exactly the same manner. The contours that correspond to none on the opposite level (for example $c_{i,6}$ or $c_{i+1,8}$) indicate additions of 0 or 2-handles (a contour is born or annihilated, respectively) and therefore their interior becomes surface of the reconstructed manifold and therefore they are triangulated.

3.3 m:n Contour Lofting

The forest mapping algorithm determined which pairs ***(A,B)*** of contour groups $A \subseteq L_i$ and $B \subseteq L_{i+1}$, represent similar regions. The next goal is to build the skin spanning the contours of ***A*** and ***B***, which are either external or internal ones but not combinations of them. This is a problem of ***m:n*** lofting, or manipulation of 0, 1 or 2-handles. Its solution, for lack of space, is left for another publication. Only the results are presented here.

4. Results and Conclusions

Fig. 6 shows results of the algorithms introduced, for both object-sampled and computer-generated data sets. These data sets present recursive cavities, and complicated transitions, in which the forest mapping algorithm has performed successfully. The implementation of similarity criterion f_2 has rendered much better results than f_1. Future work must be done in the specifics of the generalized lofting, since our strategy does not guarantee avoidance of loft self – intersections in cases of pathological groups ***A, B*** (for example, when ***A*** is ***B***, rotated by 90º). It must be noticed, however, that pathological cases are not likely, since a correct sampling interval ensures that the groups ***A*** and ***B*** have resemblance, both in shape and position. Therefore, such degenerate case would not even be admitted by the forest matching algorithm.

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