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GEOMETRIC MODELING IN DESIGN OF NAVAL ELEMENTS

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ABSTRACT

Boundary Representations (B-Reps) of actual solid parts are correct from the geometrical and topological points of view. However, when the solid to model has extreme slender ratios, the rigid rules of the B-Rep force a large number of finite elements required to model the solid interior of a closed shell (also called a 2-manifold without border). In the practice, modelling is then pursued by using only a partial shell (2-manifold with border), excluding the "interior" of it. For the same reasons, other slender elements (trusses or beams) must be modelled as 1-dimensional wires (1-manifolds with border). Assumptions are made in both cases to replace the solid model information left aside. The scenarios in which *both* 2-manifolds and 1-manifolds must coexist are undesirable from the mathematical point of view, since they render flawed topologies and geometries. However, in the engineering domain, they are required, and enabled, by replacing the information lost in the modelling with additional kinematic and structural constraints. These constraints force the 1-manifolds and 2-manifolds to intervene together in the numerical solution, therefore rendering realistic results, without actually coexisting in the geometric model. These techniques are discussed here and applied to examples of shipbuilding industry, where slender forms and extremely large models are ubiquitous.

Keywords: Geometric Modeling, Naval Elements, Mixed Manifolds

1. Introduction

Industry-related applications of Computer Aided Geometric Design (CAGD) and Computational Mechanics (and more specifically Finite Element Analysis) present challenges which are not foreseen at the layout of the CAGD and FEA formalisms.

One important issue is the one of transport and manipulation of geometric information. This issue includes data formats and primitives, in a pursue to (i) represent the actual geometry with the simplest possible geometric primitives, (ii) efficiently convey information from the CAGD side to the FEA one, without distortion, and (iii) reliably generate space decompositions (meshing) for the FEA method. In the scope of Computer Aided Engineering, these three steps occupy 80% to 90% of the resources, with the remaining balance being filled by the actual Computational Mechanics Problem (boundary conditions, pre-processing, numerical solution and post-processing).
In the realm of naval (as well as aircraft) applications an additional difficulty is presented by the slenderness ratio of the elements. They are thin and long. If these elements are represented by solid models, their slenderness or shape ratio causes an explosive growth in the number of analysis elements which compose the solid. The characteristic of computational explosion in CAE when using solid models is transversal to all the known methods (including Fixed Grid ones [GARCIA.2001]). The natural reaction is, obviously, to model with 2-manifolds (called sometimes SHELLs) or with 1-manifolds (BEAM or WIRE). This action, however, leads to scenarios in which 1- and 2-manifolds must co-exist, for example when stiffeners are used to reinforce a shell. This situation is foreign to formal topology, and yet normal in engineering. The coexistence of 1- and 2-manifolds in FEA, made possible by kinematic and structural considerations complementing or replacing the topological ones, is the scope of this article. This coexistence is presented here in the scope of shipbuilding.
Section 2. discusses the literature relevant to the subjects and draws conclusions about the actions to take to overcome the difficulties. Section 3 describes alternatives to attack the problems and examines dramatic modeling failures caused by ignoring the differences in topological dimensions. Successful results as well as examples of are discussed in Section 4, while conclusions are drawn in Section 5.

2. Literature Survey

A precise definition on what a *shell* or a *wire* are is required. Formal synonymous are 2-manifold and 1-manifold, respectively, embedded in $\boldsymbol{R^3}$.
Definition of k-Manifold. A *k*-manifold $\boldsymbol{M}$ embedded in $\boldsymbol{R^m}$ is a set of points $\boldsymbol{p}$ in $\boldsymbol{R^m}$ such that every ball $\boldsymbol{B(p, \delta)}$ of radius $\delta < \delta_p$ (δ_p a small enough radius depending on $\boldsymbol{p}$) centered in $\boldsymbol{p}$ intersects $\boldsymbol{M}$ in a set of points isomorphic to a unit disk in $\boldsymbol{R^k}$. In what follows we make m=3 (i.e $\boldsymbol{R^3}$) (see [Morse.1934, Fomenko.1997]).
Definition. Shell. A "*shell*" (Figure 1) is mathematically defined as a 2-manifold $\boldsymbol{M}$ embedded in $\boldsymbol{R^3}$. Informally, this means that every ball $\boldsymbol{B(p,r)}$ of small enough radius r, centered in $\boldsymbol{p}$ ($\boldsymbol{p}$ being a point of set $\boldsymbol{M}$) intersects $\boldsymbol{M}$ in a set $\boldsymbol{D}$ which is isomorphic with a unit planar disc in $\boldsymbol{R^2}$ (an example appears in Figure 3).
Definition. Wire or Beam. A "*wire*" or "*beam*" (Figure 2) is mathematically defined as a 1-manifold $\boldsymbol{M}$ embedded in $\boldsymbol{E^3}$. Informally, this means that every ball $\boldsymbol{B(p,r)}$ of small enough radius $\boldsymbol{r}$, centered in $\boldsymbol{p}$ ($\boldsymbol{p}$ being a point of set $\boldsymbol{M}$) intersects $\boldsymbol{M}$ in a set $\boldsymbol{D}$ which is isomorphic with the interval [-1,1], which is called a "unit planar disc in $\boldsymbol{R^1}$".

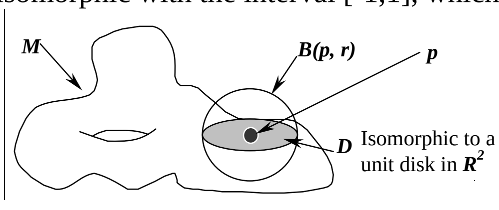


Figure 1 . A 2-Manifold in R3.

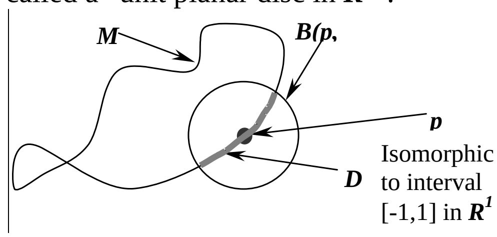


Figure 2. A 1-Manifold in R3.

2.1 General Boundary Representation (B-rep)

The Boundary Representation (B-rep) is the convention that a body is univocally expressed by its boundary ***M***, which is a 2-manifold in $\boldsymbol{R}^3$ (see [Mantÿla.1988]). For that, it is necessary to specify which is the "interior" of ***M***. When ***M*** is not completely closed, there are points called *boundary* or *border* points on ***M*** for which the disc ***D*** mentioned above is homeomorphic to a semi-disc instead than to a full disc. In such a case it is said that ***M*** is a *2-manifold with border, embedded in* $\boldsymbol{E}^3$ (Figure 4). Notice that if ***M*** has borders, it is impossible to define interior vs. exterior. On the other hand, 2-manifolds with boundary are essential to define non closed shells , of vital importance in applications of machining CNC, stereolitography, visualization. Additionally, Finite Element Analysis software (FEA) usually requires shell data rather (with borders) than a solid object.

The B-rep schemes require a strict hierarchy of geometric and topologic entities. Although every geometric modeller (ACIS, ParaSolids, IDEAS, CATIA, etc.) uses different names, a typical hierarchy is shown in Table 1.

Table 1. Relations and hierarchy of topologic and geometric elements in B- rep (from [Ruiz.2002a])

TOPOLOGIES		**GEOMETRIES**	
BODY	Set of possibly disconnected solid regions or LUMPs		
LUMP	A solid connected region, bounded by SHELLs		
SHELL	A connected part of the boundary of a LUMP region. A 2-manifold in general without border.		
FACE	A connected subset of points belonging to one SURFACE. The subset is bounded by closed contours (loop) formed by EDGES (edge) contained in the SURFACE.	**SURFACE**	Analytic surface, in parametric form $[X(u,v),Y(u,v),Z(u,v)]$ or implicit $f(x,y,z)=c$.
LOOP	Closed non-autointersecting path, formed by EDGES and fully contained in a SURFACE carrier.		
EDGE	A connected subset of points belonging to a CURVE. Two VERTEX, contained in the CURVE bound the subset.	**CURVE**	Analytic curve, in parametric form **[X(u),Y(u) ,Z(u)]** or implicit **f(x,y,z)=c**.
VERTEX	A connected subset of points belonging to a POINT. Obviously there is only one POINT in such a subset.	**POINT**	(x,y,z) in $\boldsymbol{R}^3$

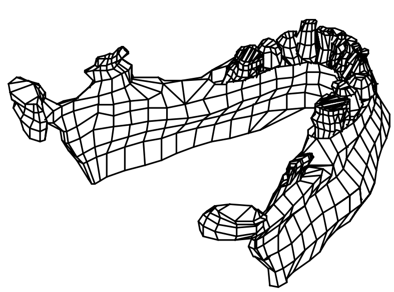


Figure 3. SHELL (2-mainfold) without border (from [Ruiz.1999])

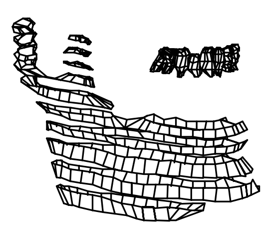


Figure 4. Several SHELLs (2-Manifolds) with border (from [Ruiz.1999])

2.2 Finite Element Analysis and Boundary Representation (B-rep)

Figure 5 presents a typical naval element, which is a plate with stiffeners. The model presented is a full B-rep one, which is also exploded. This construct, while recognized by FEA software, presents the problem of explosive number of elements at meshing time, given its large wide/thickness or length/thickness ratios. Since the dimensions of the elements are limited by its small thickness, many TETRA or BRICK elements are required to fill its space, therefore placing an unacceptable computational burden.

As a result, main deviations from B-Rep are produced: (i) objects which are slender or thin solids in real world are abstracted in FEA as pairs (SHELL, thickness) or (BEAM, area), causing a major departure from the original B-Rep scheme, and (ii) manifolds with differing dimension 1- , 2- or 3- share the space $\boldsymbol{R}^3$ in the abstraction of a physical situation.

The boolean union of a 1- and a 2- manifolds (or any union of differing dimension manifolds) violates the fundamental premises of a B-rep, causing illegal constructs in topology, called "dangling" FACES. Figure 6 and Figure 7 show two examples of FEA constructs, which are illegal from the point of view of topology and B-rep., and yet they are approximations of very common situations in engineering practice.

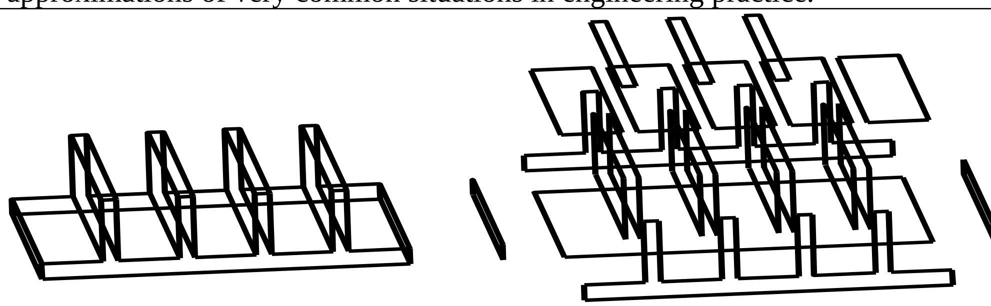


Figure 5. Lower: Plate with stiffeners modeled as a full B-rep solid. Upper: exploded B-Rep

In Figure 6 stiffeners are placed on a thin plate. Since the stiffeners are modeled as 1-manifolds, geometrically they should be either embedded in the plate, thus having no effect, or removed and parallel to it, therefore contributing in nothing to the plate stiffness. In either case, other assumptions are required to correctly model their position and effects on the plate. In Figure 7 two manifolds are joined (welded), but the abstraction of the situation in not a manifold, since non-manifold EDGEs appear, in which 3 FACEs are incident. Manifold property requires that on a particular EDGE, 1 or 2 FACEs be incident. And yet, the abstractions in Figure 6 and Figure 7 correspond to actual engineering situations. The challenge is, to represent them by selectively allowing non-manifold situations, and correcting them by adding conventions and constraints which fill the topological vacuum created.

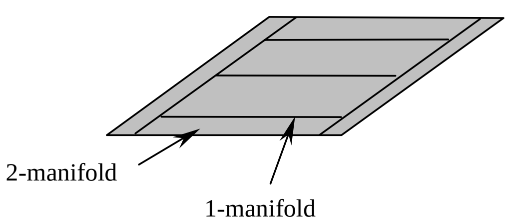


Figure 6. Plate (2-manifolds) with Stiffeners (1-manifolds).

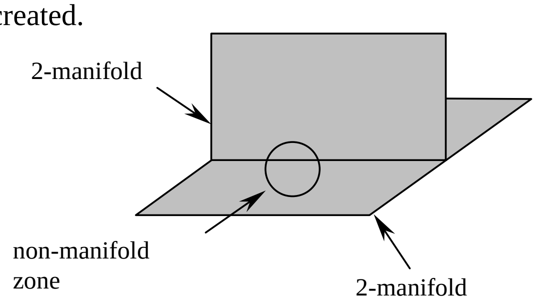


Figure 7. Two 2-manifolds assembled in a non-manifold topology.

3. Methodology

The topological problem considered is: given a 2-manifold which represents a thin plate, and a set of 1-manifolds welded on it, and representing stiffeners (Figure 5), one must model the stiffening effect of the 1-manifolds by either (i) setting plausible boundary conditions of the contact surface or (ii) conceptually linking the degrees of freedom of the elements in the manifolds, to model the restrictions that they impose on each other, therefore producing a stiffening effect.

3.1 Modelling by Manipulation of Boundary Conditions on 2-manifolds.

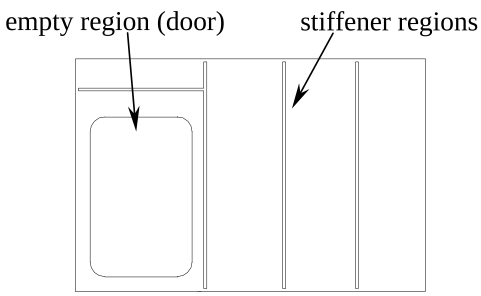


Figure 8. Wall with Stiffeners (Mamparo, Baos).

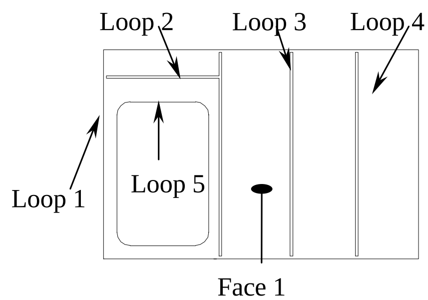


Figure 9. Topological Division of Face with Loops.

As said before, modeling the wall and stiffeners in Figure 8 with full solid models leads to explosive computational expenses. Therefore, it is necessary to model using 1- and 2-manifolds (to get elements SHELLS and or BEAM). Figure 9 shows the topological model using only 2D constructs (2-manifolds). The wall is modeled as a FACE, with one outermost LOOP, and one internal one, to model the door. However, because of requirements in the FEA software to exactly locate the boundary conditions, one must define additional LOOPs, which limit the regions in which the stiffeners are welded. These regions are assigned a null displacement to model the problem. Results of the modeling with this combination of topological subdivision and boundary conditions are shown in Figure 12 and Figure 13. As seen, the zero displacement dictated for the stiffener areas represents an exaggerate forecast for their role, although the results may seem plausible.

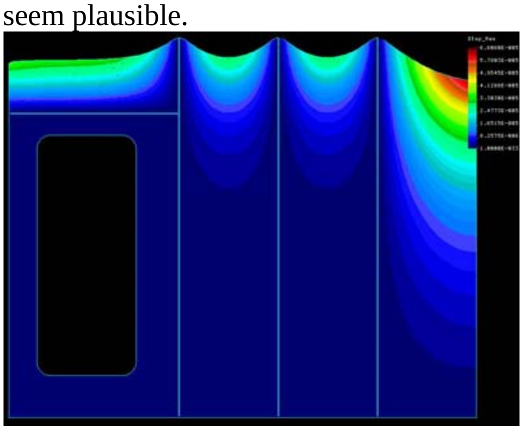


Figure 10. Deformation (scale 4995.44) of Plate + Stiffeners model using constraints on topological regions.

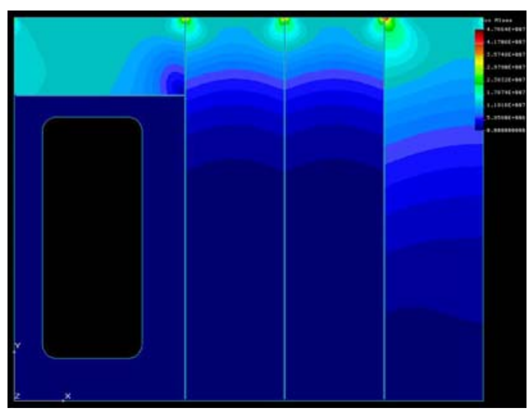


Figure 11. Von Misses stress of Plate + Stiffeners model using constraints on topological regions.

3.2 Modelling by Perpendicularly placed 2-manifolds.

In an attempt to handle solid geometry and topology (Figure 5), the Face 1 (in Figure 9) and FACEs of the stiffeners perpendicular to Face 1 were extracted from the solid model Plate+Stiffeners, and meshed. This modeling basically corresponds to flat SHELLs placed perpendicular to each other, attempting to stiffen the resulting structure. In the computer prediction the stiffener areas actually do nothing to arrest buckling in the modeling since those areas are truly separate constructs with respect to the wall body. Therefore, a collapse is inaccurately predicted (Figure 12 and Figure 13). Although the meshing in the perpendicular shells was not compatible, the problem here stems in a more fundamental way in the fact that the 2-manifolds or SHELLs do not present inertial moments, or stiffness against bending.

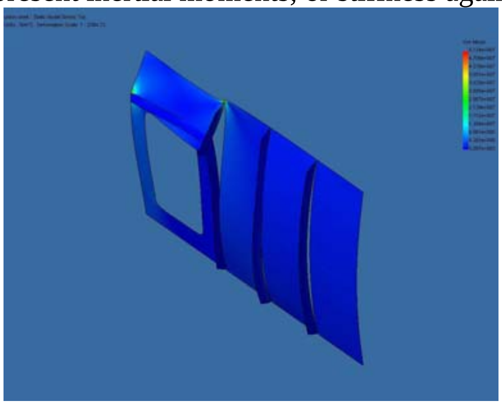


Figure 12. Deformation. Modeling with perpendicular 2-manifolds (SHELLs) extracted from full solid model.

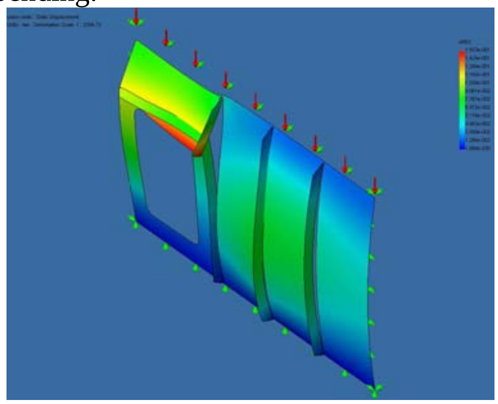


Figure 13. Von Misses stress. Modeling with perpendicular 2-manifolds (SHELLs) extracted from full solid model.

3.3 Modelling by Cancellation of Degrees of Freedom.

The cancellation of degrees of freedom is based on the following steps or assumptions:

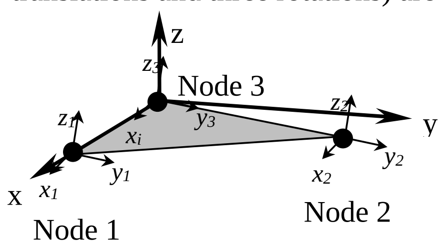
(i) Elements of different topology coexist in the scenario. In this case, one has SHELL (2-manifold) and BEAM (1-manifold) elements. These elements are in contact.

(ii) SHELL elements are equipped with a constant thickness *t* and BEAM elements are equipped with a cross sectional area A_s.

(iii) Nodes for Finite Element Analysis have a private coordinate system, with, in general 6 degrees of freedom (three translations and three rotations)

(iv) For simplicity, SHELL elements are here assumed to have 3 nodes (they may have 4, 5 or more), with six degrees of freedom per node (three translations and three rotations) for structural analysis. They have a coordinate frame (x-y-z) as per Figure 14.

(v) For simplicity, a BEAM element is assumed to have a 2 nodes for purposes of three-dimensional nonlinear uni - axial structural models. Each element owns a coordinate frame (x-y-z) as per Figure 15. Six degrees of freedom (three translations and three rotations) are considered for each node.



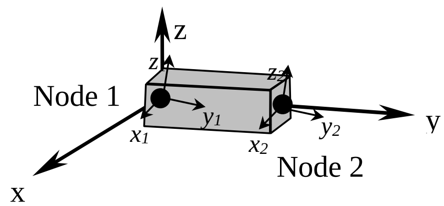


Figure 14. Coordinate frame of SHELL element with node coordinate frames and degrees of freedom.

Figure 15. Coordinate frame of BEAM element with node coordinate frames and degrees of freedom

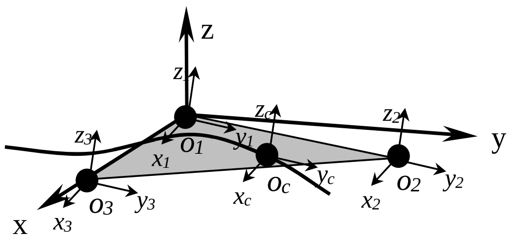


Figure 16. A triangular element(SHELL) on a 2-manifold carrying a BEAM element of a 1-manifold. Initial conditions

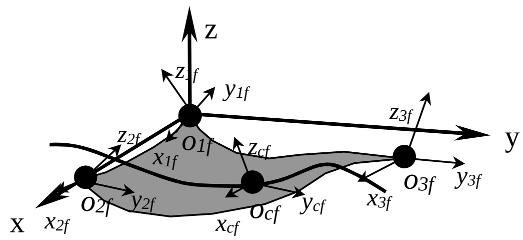


Figure 17. Deformed triangular elements SHELL and BEAM by enforcing position and orientation constraints on nodes.

Figure 16 shows the initial disposition of a 2D mesh triangular element on which a 1D element is resting. The enforcement of unity between the plate and the beam is done not through node coincidence, but through additional kinematic constraints to be satisfied (see [Cook.1989]). These constraints (in their simplest form) are (see [Ruiz.1996] and Figure 17):

(i) ***f( )*** is the deformation of the initial triangle in its final shape:

$$f(O_1) = O_{1f}\ ;\ f(O_2) = O_{2f}\ ;\ f(O_3) = O_{3f}$$

(ii) The final Coordinate System [xc_f, yc_f, zc_f, Oc_f] of the beam node must rest on the deformed triangular element $P_f(u,v)$: $Oc_f \in P_f(u,v)$

(iii) The Z_{cf} vector (in the deformed configuration) of the beam must be normal to the deformed surface at the Oc_f point:

$$\nabla P_f(Oc_f) = Zc_f$$

(iv) The relative position of the beam node within the triangle must be maintained at the final position: $f(O_c) = Oc_f$

(v) The strength of the beam is related to its deformations and force and torque interpolated at the node points:

$$F(O_c) = K_f . \Delta_L ([x_c, y_c, z_c, O_c],[x_{cf}, y_{cf}, z_{cf}, O_{cf}])$$

$$T(O_c) = K_\tau . \Delta_\Theta ([x_c, y_c, z_c, O_c],[x_{cf}, y_{cf}, z_{cf}, O_{cf}])$$

Because the mathematical statement of the Finite Elements, it is usually not ensured that the curve representing the deformed beam rests on the deformed patch $P_f(u,v)$. Only at the checkpoints (the node O_c) is this required. The actual shape of the deformed SHELL element and the BEAM one is to be defined by the basis functions of the analysis, and, although for display purposes the elements look like triangles in the deformed situation, this may not be the case (see [Braess.1997]). The mentioned constraints (and the structural ones), are added to the numerical problem to solve.

4. Results

By taking advantage of the compactness of 1-manifold and 2-manifold models, possibly combined as discussed in previous sections, larger models may be analyzed. Here we present the results of the modeling and analysis of the Plate+Stiffeners model (see Figure 18 and Figure 19). The reader may observe that in effect, the constrained degrees of freedom convey to the model the presence of the stiffeners, therefore achieving a more realistic analysis as compared with Figure 10 to Figure 13.

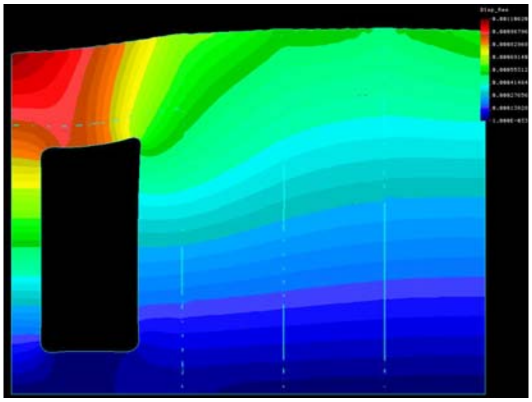


Figure 18. Plate plus Stiffeners model using Bonding of Degrees of Freedom. Deformation (scale 298.3).

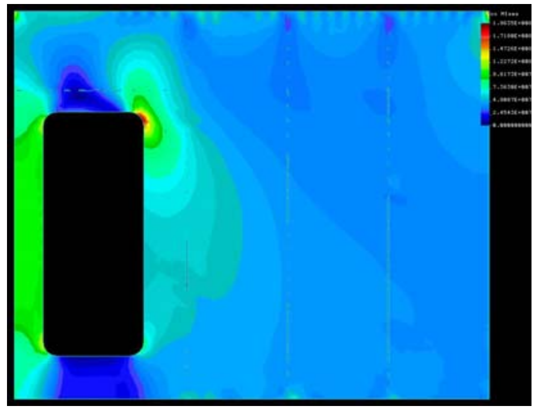


Figure 19. Plate plus Stiffeners model using Bonding of Degrees of Freedom. Von Misses stress.

In similar way, by using 2-manifold elements the ship hull was modeled. The CAD stage of the geometric modeling was done by using at different times both, facetted and NURBS models. In the first case, a Piecewise Linear (PL) continuous shell is achieved, formed by flat facets, which may be used for additional meshing or used as they come. In the other case, the facetting is preformed on a NURB parametric surface set. In either case, no solid elements where used. The modeling is partial in the sense that no stiffening coming from the super- structure was considered yet. A dynamic analysis was conducted in order to establish the most important vibration modes of the ship hull are, as it crosses a wave, and most of the hull is in the air, with no support.

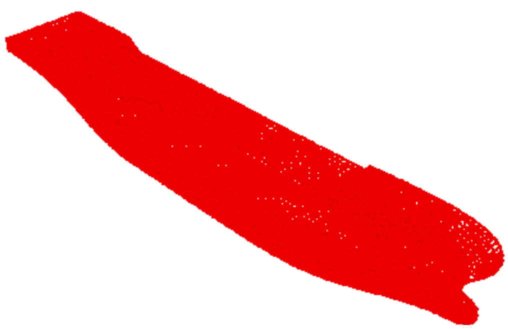


Figure 20. Visualization of Ship Hull SHELL elements. Facetting performed at the CAD software. STL format.

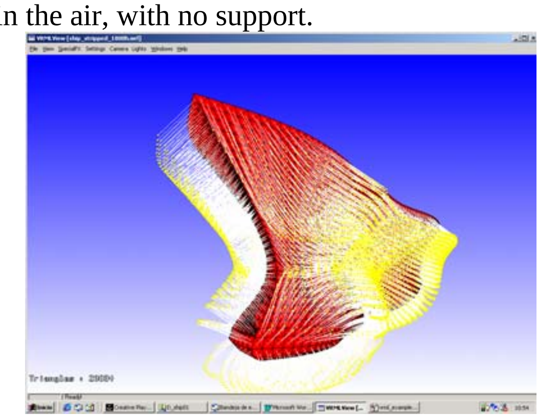
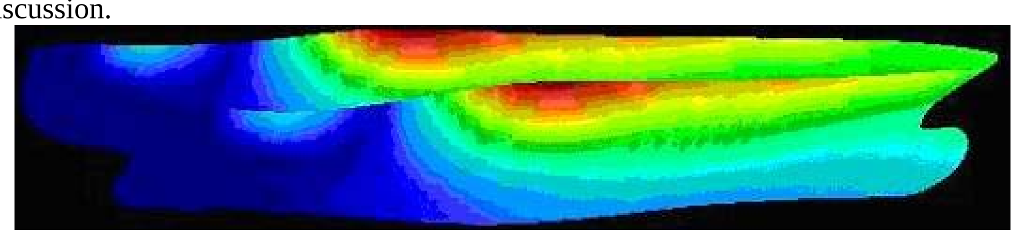


Figure 21. VRML visualization of Ship hull for normal vector monitoring and correction.

Figure 22 shows the 5 largest vibration modes determined. The analysis was performed assuming that the stern of the ship is fixed in cantilever conditions. Although this is not exactly the situation in service conditions, it served as display scenario for the present discussion.



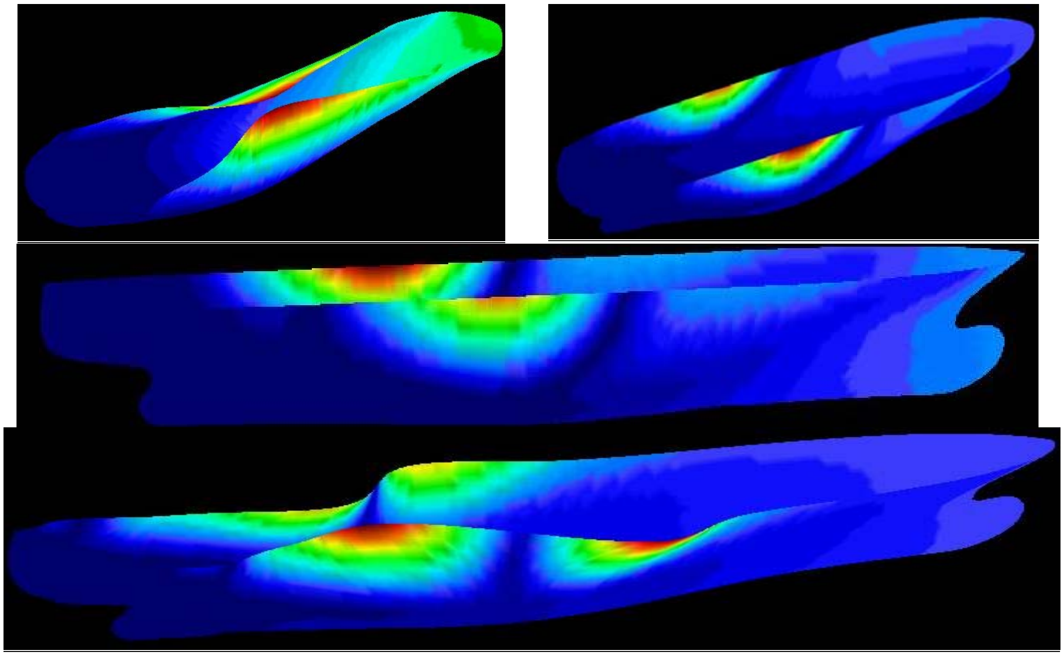


Figure 22. First 5 modes of vibration for Ship hull model.

5. Conclusions

The results for the plate - stiffeners in Figure 18 and Figure 19 and may be positively compared to those in Figure 10 and Figure 11 in which the effect of the stiffeners is overstated. In the same manner, it is better as the one in Figure 12 and Figure 13, where the stiffening basically does not exist. As said before, modeling plate and stiffeners by allowing only elements of the same dimensionality leads, along with limitations of the software used, to wrong results.

On the other hand, it must be understood that mixing manifolds of different dimensionality makes no sense in the context of geometric modeling, and whatever usage made of this mixture must lead to incorrect outcomes. However, at the level of practice in engineering analysis, additional kinematic and structural conditions are used to relate the differing dimensionality manifolds at the level of numerical solutions, while keeping them in different spaces at the level of geometric modeling.

The usage of kinematic and structural conditions its itself an approximation, which needs to be exercised with care. For example, the model examined does not enforce the permanence of the 1-manifold (beam) on the 2-manifold (shell) except for a finite number of points (the nodes on the beam). In other neighborhoods the numerical analysis shows a separation and / or invasion of one in the other. Finally, it should be kept in mind that, since a physical beam has area and inertia, in reality its center of gravity is removed from the plate it rests on. Therefore, the abstraction of embedding the 1-manifold in the 2-manifold is only a graphic user interface approximation of the problem. It is different from the problem stated at the equation level, which is considered the correct one for the numerical precision and assumptions applied.

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