HexFlex Mechanism Modeling by Design of Computer Experiments

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Abstract

Compliant mechanisms are an instance of mechanical devices designed to transfer or transmit motion, force, or energy from specified input ports to output ports by elastic deformation of at least one of its members. The main advantage of compliant mechanisms with respect to traditional rigid-link mechanism is that fewer parts, fewer assembly process and no lubrication are required. The HexFlex is a parallel compliant mechanism for nano-manipulating that allows six degrees of freedom of its moving stage. This mechanism was designed for high precision an repeatability. This article presents a methodology to model compliant mechanisms behavior under quasi-static conditions using computer experiments, reducing costs of experimentation of product development. The methodology is used to establish a mathematical model that relates the actuator forces at the input ports with the position and orientation the end-effector stage of the Hexflex. This mathematical model has direct application in model-based control as an advantage with respect to other models, *e.g.* Finite Element Method. The mathematical model of the HexFlex is achieved using metamodels. The term methamodel is used to represent a simplified and efficient mathematical model of unknown phenomenon or computer codes. The metamodel of the HexFlex is performed from virtual analyses made using the Finite Element Method (FEM). Simulations of the metamodel were made founding good accuracy with respect to the virtual experiments.

keywords: Design of experiments, Metamodeling, Compliant Mechanisms, Plackett-Burman Design, Factor Screening.

1 Introduction

Compliant mechanisms are an instance of mechanical devices designed to transfer or transmit motion, force, or energy from specified input ports to output ports by elastic deformation of at least one of its members. The main advantage of compliant mechanisms with respect to traditional rigid-link mechanism is that fewer parts, fewer assembly process and no lubrication are required [1]. Due to the complexity of their motion, compliant mechanisms are difficult to design and analyze by traditional kinematic methods [2].

Modeling the behavior of compliant mechanisms is required to accurately design and model-based control. Currently, the analysis and design methods of compliant mechanisms can be categorized as kinematics-based approach and continuum-based approach [3]. The kinematics-based approach considers a compliant mechanism as a traditional mechanism where the joints are replaced by torsional and linear springs [4, 5, 6]. This modeling is restricted to simplified compliant mechanism geometries with lumped compliance. The continuum-based approach generates compliant mechanisms topology, shape and size from a single piece of material using topology optimization for given deformation requirements [7, 8, 9]; this approach reduces human intervention in the design giving as a result structures that can be impossible to build. This modeling is limited to the design process requiring additional modeling for motion control.

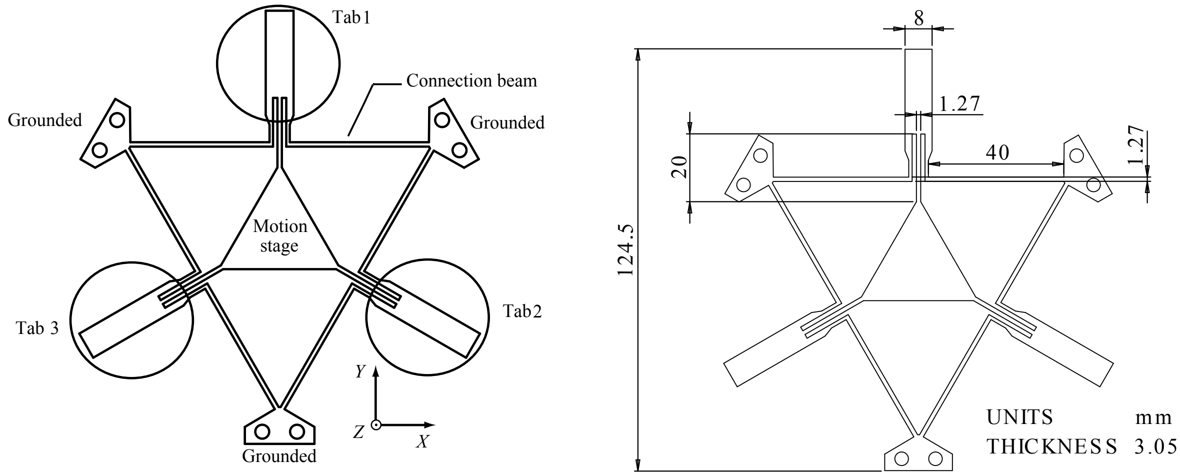
This article presents a methodology to model compliant mechanisms behavior under quasi-static conditions using computer experiments, reducing costs of experimentation of product development. The methodology allows to find a mathematical model of the mechanism that has direct application in model-based control as an advantage with respect to other models, *e.g.* Finite Element Method. This modeling is useful for mechanisms with lumped or distributed compliance. The term metamodel represents a surrogate model and is based on the use of statistical techniques to yield mathematical equations that approximate the results rendered by computer algorithms such as Finite Elements Method (FEM) [10]. Metamodels have benefits in variable screening, reducing design costs and design optimization [11].

The metamodeling is applied to model the quasi-static behavior of the HexFlex mechanism. The HexFlex is a six degrees of freedom parallel compliant mechanism with distributed compliance for nano-manipulating designed by Martin L. Culppeper and Gordon Anderson [12, 13].

The topology and dimensions of the HexFlex are shown in Fig. 1. This mechanism allows the motion stage translation and rotation trough the X, Y and Z axes. The HexFlex is composed by a triangular motion stage, three tabs to provide an interface with the actuators, and six connection beams between the motion stage and the grounded zone, Fig. 1(a).

To control the motion stage there are two actuators in the external edge of each tab. For each tab, one actuator acts in direction parallel to the connection beams (called direction one and denoted $D1$) and, the other actuator acts perpendicular to the tab (in Z direction, called direction two and denoted $D2$), Fig.2. Tabs are denoted T1, T2, T3. The motion of an specific actuator is denoted by the tab followed by the direction using the convention shown in Fig.2.

The actuators used in the experiments allows a force of ± 1 N and displacement of ± 2 mm. Slowly varying in time forces are assumed for the experiments (quasi-static experiments). Planar



(a) HexFlex components (b) HexFlex main dimensions

Figure 1: Six degrees of freedom compliant mechanism [14]

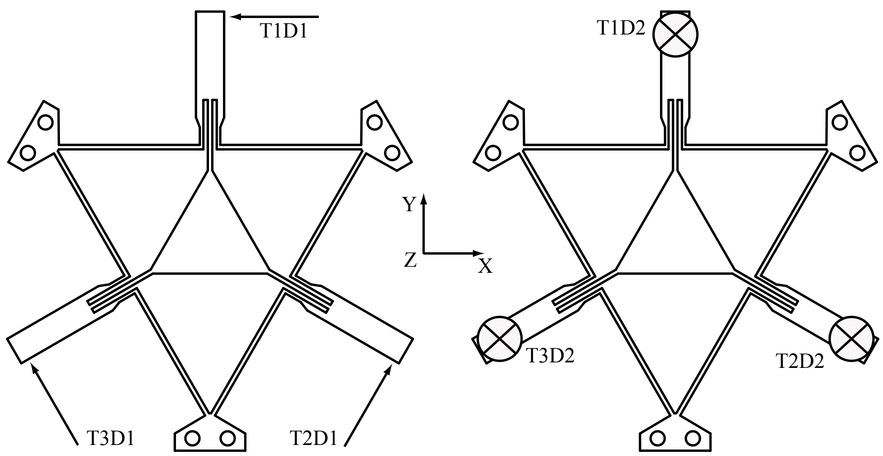


Figure 2: *HexFlex actuators direction*

and non-planar displacements may be made simultaneously. The material selected to model the mechanism is Aluminum 7075.

2 Metamodeling of complaint mechanisms methodology

With Metamodeling of compliant mechanisms we are looking for a function that relates the input forces and torques (τ) with translations and rotations of the end-effector ($\mathbf{r}$) under quasi-static conditions:

$$f : \tau \rightarrow \mathbf{r} \quad (1)$$

where $\tau = \begin{bmatrix} \tau_1 & \tau_2 & \cdots & \tau_n \end{bmatrix}^T$ and $\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \cdots & r_m \end{bmatrix}^T$ with $m \leq n$. For an end-effector taking and arbitrary pose, we have $m = 6$. We assume that mechanisms are not redundant, then $m = n$.

To modeling compliant mechanisms under quasi-static condition using computer-based metamodel from computational experiments, the methodology presented in Fig. 3 is proposed.

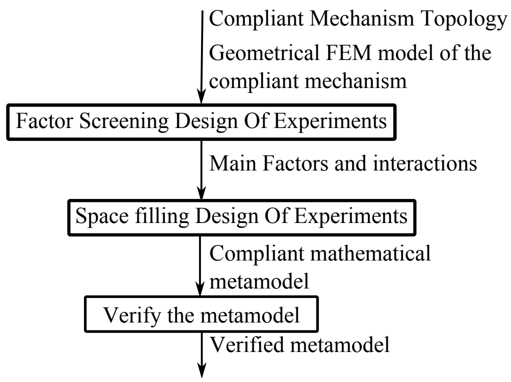


Figure 3: *Methodology for analysis of compliant mechanisms*

The metamodeling methodology for compliant mechanisms is summarized as:

1. Define the compliant mechanism topology and determine how to actuate it.
2. Perform a geometrical FEM model of the compliant mechanism.
3. Use a factorial Design Of Experiments (DOE) (*e.g.* Plackett Burman) to screen variables.
4. Use an Space Filling Design of Experiments (*e.g.* Uniform Design [15]) to fine tune the mathematical model of the mechanism.
5. Perform computer experiments, and construct the surrogate model of the kinematics of the compliant mechanism.
6. Verify the accuracy of the metamodel using extra experiments [10].

In section 3 the proposed methodology is applied to obtain a mathematical metamodel of the HexFlex parallel compliant mechanism. The developed metamodel relates the actuator forces at the tabs with the position and orientation of the end-effector stage.

3 Metamodeling of the HexFlex parallel compliant mechanism

The HexFlex topology, functioning and main dimensions are described in sec. 1. To define the metamodel function, the vector of input forces (τ) and pose of the end-effector ((r)) are definied by:

$$\tau = \begin{bmatrix} T1D1 & T1D2 & T2D1 & T2D2 & T3D1 & T3D2 \end{bmatrix}^T \tag{2}$$

$$\mathbf{r} = \begin{bmatrix} x & y & z & \theta_x & \theta_y & \theta_z \end{bmatrix}^T \tag{3}$$

where the end effector pose (position and orientation) is defined by three translational components (x, y, z) and three differential orthogonal rotations (θ_x, θ_y, θ_z), and the input forces correspond to the description made in sec. 1. The reference frame is assumed to be coincident with the center of the motion stage in its relaxed position, Fig. 4

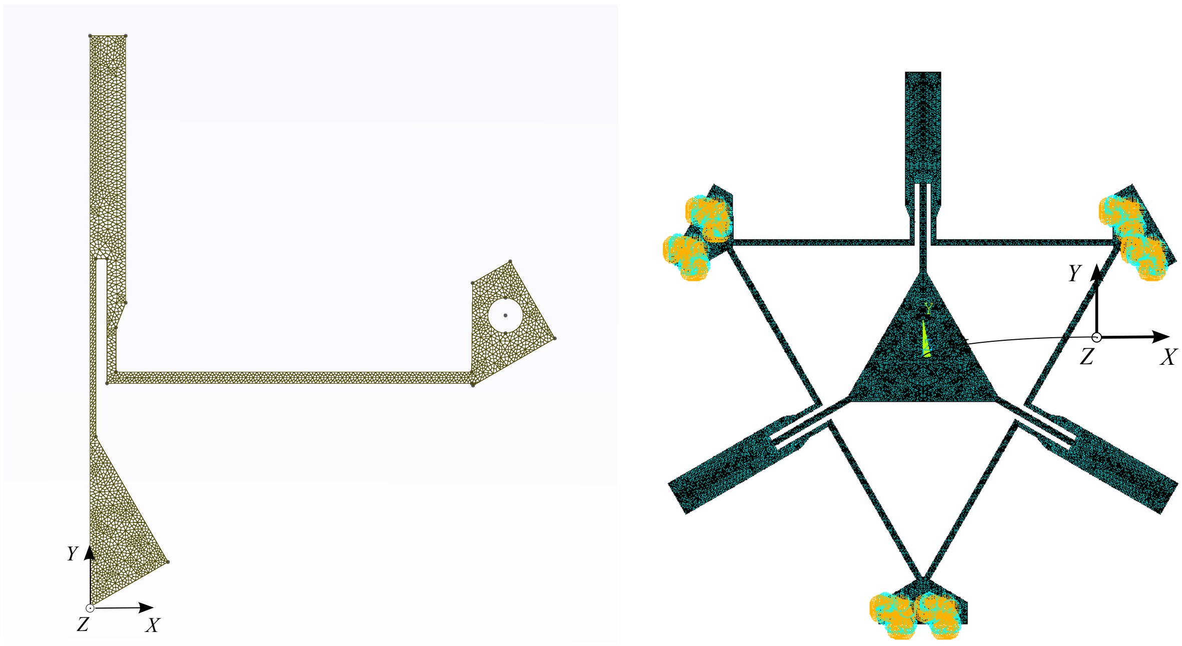
Using the symmetry of the mechanism and the dimension shown on Fig.1(b), a quarter part of the mechanism was modeled and meshed to made a geometric FEM model of the mechanism, Fig. 4(a). Using geometric transformations, the mechanism was completed developing a symmetrical mesh. Then the mesh was exported to ANSYS to run the virtual design of experiments, Fig. 4(b). The computer experiments consist in given a set of input load in the tabs,to obtain the position and orientation of the reference frame on the mechanism.

Factor	**Low level**	**High level**
T1D1	−1N	+1N
T1D2	−1N	+1N
T2D1	−1N	+1N
T2D2	−1N	+1N
T3D1	−1N	+1N
T3D2	−1N	+1N

Table 1: Studied Factors. Forces in Tabs of the HexFlex

In Table 1 the the high and low level of each factor are displayed. The factors or inputs of the experiments are defined by Eq. 3.

In sections 3.1 and 3.2, the Factorial and Space Filling Design of experiments to define the metamodel of the HexFlex.



(a) Quarter mesh (b) FEM model in ANSYS

Figure 4: FEM model of the HexFlex

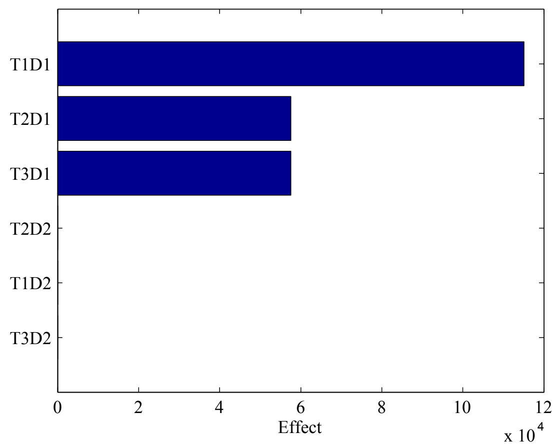
3.1 Fractional Factorial Design of Experiments

To screen variables a Plackett Burman DOE [16, 17] with 12 runs is made. A script was developed to automatically generate the virtual experiments and its results. The design of experiments matrix and the results of each response are show on Table 2.

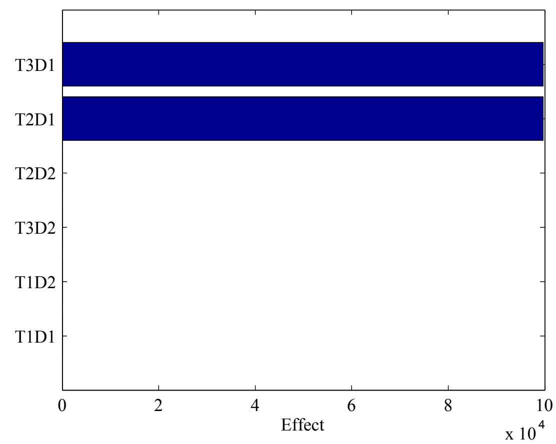
To analyze the results of the Plackett Burman DOE, Pareto (Fig.5) and Half Normal Probability (HNP) plots are made (Fig.6). These analysis provide a simple way to examine the response variables (*i.e.* x, y, z, etc) and the relative importance of the factors and interactions of the experiment.

The Pareto charts results coincide with Half Normal Probability (HNP) showing that the main interactions are consequent with the topology of the mechanism; and also, that the inputs (actuator forces) in the mechanism are influenced by its symmetries and, for that reason some effects has the same value. The main effects for each response are summarized in table 3.

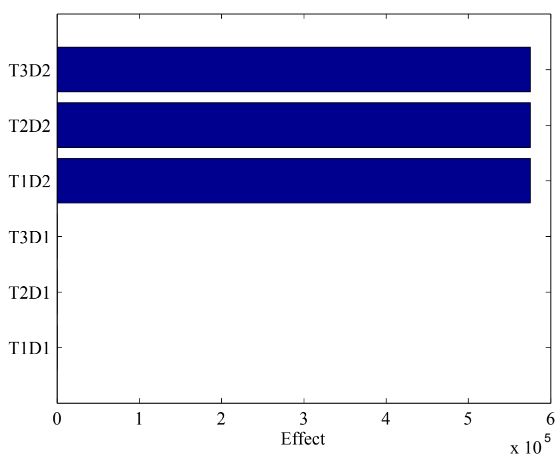
In-plane displacements (x, y, θ_z) are generated when actuators acts in direction one (D1); and out-of-plane displacements (z, θ_x, θ_y)are generated when actuators acts in direction two (D2).



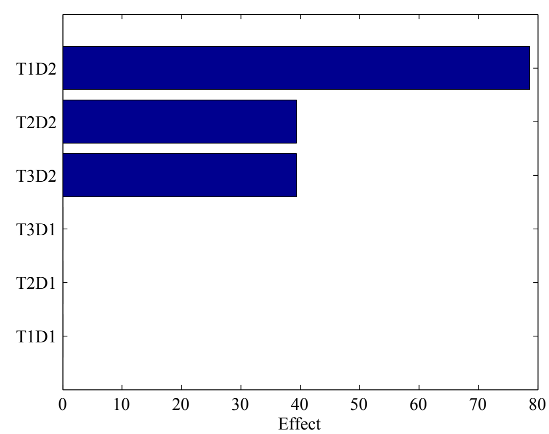
(a) Pareto Effects for x



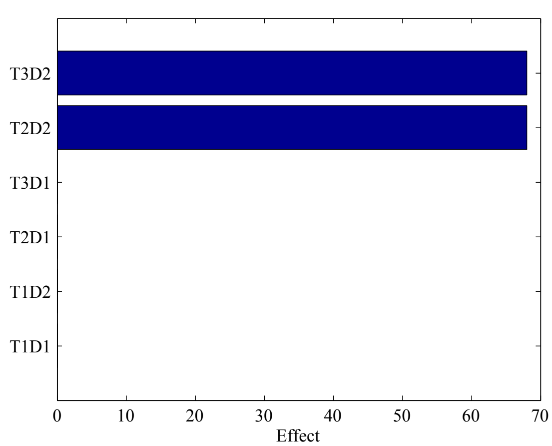
(b) Pareto Effects for y



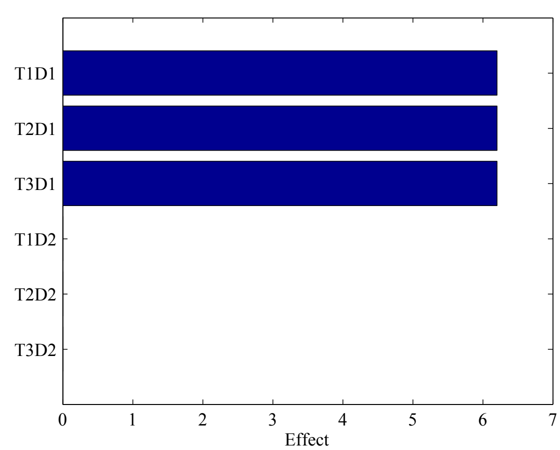
(c) Pareto Effects for z



(d) Pareto Effects for θ_x

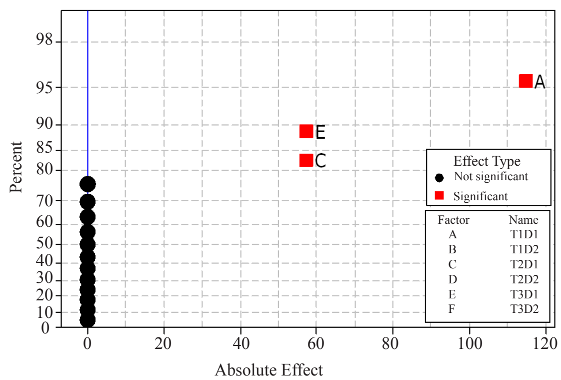


(e) Pareto Effects for θ_y

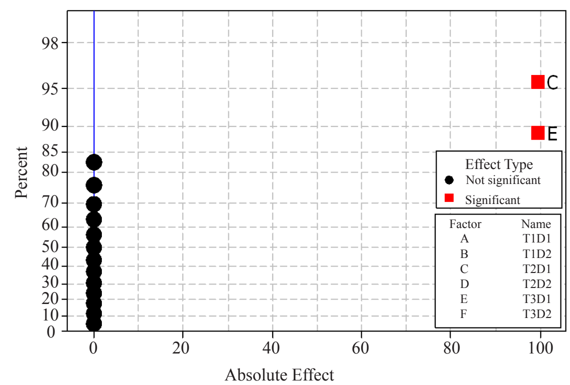


(f) Pareto Effects for θ_z

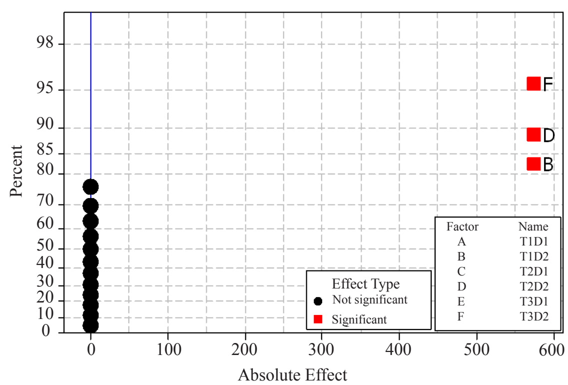
Figure 5: Pareto Charts. Placket Burman DOE for 12 runs and 6 factors for HexFlex quasi-static conditions



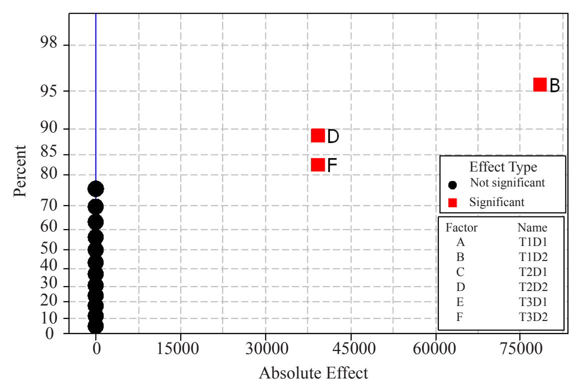
(a) Half Normal Effects for x



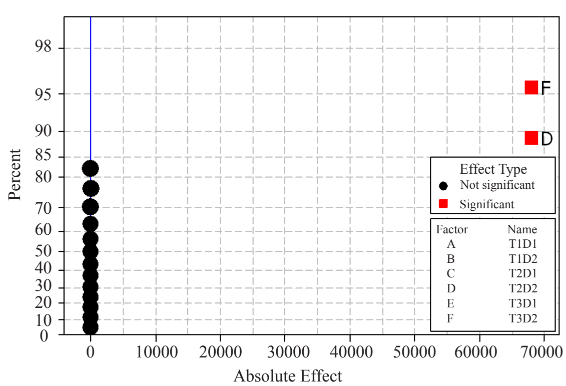
(b) Half Normal Effects for y



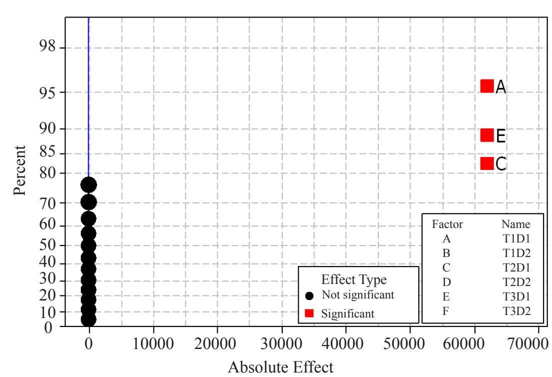
(c) Half Normal Effects for z



(d) Half Normal Effects for θ_x



(e) Half Normal Effects for θ_y



(f) Half Normal Effects for θ_z

Figure 6: Half Normal Probability Plots. Placket Burman DOE for 12 runs and 6 factors for HexFlex quasi-static conditions

Design Matrix						Responses					
T1D1 [N]	T1D2 [N]	T2D1 [N]	T2D2 [N]	T3D1 [N]	T3D2 [N]	x [μm]	y [μm]	z [μm]	θ_x [μrad]	θ_y [μrad]	θ_z μrad
1	−1	1	−1	−1	−1	115056	0, 6	−862976	0, 0001	−0, 0001	3, 10176
−1	−1	1	1	1	−1	−57529	99636, 5	−287659	−39, 3596	−68, 0656	3, 10183
1	−1	−1	−1	1	1	−3	−0, 6	−287655	−39, 2665	68, 1194	−9, 30545
−1	−1	−1	−1	−1	−1	−57525	−99636, 5	−862976	0, 0001	−0, 0001	3, 10186
1	1	−1	1	1	−1	−3	−0, 6	287655	39, 2665	−68, 1194	−9, 30545
1	1	1	−1	1	1	57525	99636, 5	287659	39, 3596	68, 0656	−3, 10186
−1	−1	−1	1	1	1	−115056	−0, 6	287662	−78, 6262	0, 0539	−3, 10176
−1	1	−1	−1	−1	1	−57525	−99636, 5	287659	39, 3596	68, 0656	3, 10186
−1	1	1	1	−1	1	3	0, 6	862976	−0, 0001	0, 0001	9, 30545
−1	1	1	−1	1	−1	−57529	99636, 5	−287662	78, 6262	−0, 0539	3, 10183
1	1	−1	1	−1	−1	57529	−99636, 5	287655	39, 2665	−68, 1194	−3, 10183
1	−1	1	1	−1	1	115056	0, 6	287662	−78, 6262	0, 0539	3, 10176

Table 2: Plackett-Burman DOE Matrix for Six factors and 12 runs

From the HPN and Pareto plots it is evident that there are factor with the same effect in each corresponding response; fact that evidence that there are symmetries in the way to act the mechanism to achieve desired movement, which is consistent with the triangular topology of the mechanism.

The fact that interactions are not important in the behavior of the mechanism, and that the HexFlex is actuated in a quasi-static state, evidence that the mechanism is well-behaved and do not present large nonlinearities. The behavior of the HexFlex could be modeled using low-order polynomials.

3.2 Metamode of the HexFlex

To generate a polynomial model an Uniform DOE [18] with six factors and six levels is used. The design matrix and the responses results found using Ansys are shown on Table 4. The polynomial model is shown in Eq.4

Response	**Main Factors [N]**
x	T1D1, T2D1, T3D1
y	T2D1, T3D1
z	T1D2, T2D2, T3D2
θ_X	T1D2, T2D2, T3D2
θ_Y	T2D2, T3D2,
θ_Z	T1D1, T2D1, T3D1

Table 3: Summary of Half-Normal Probability and Pareto plots for HexFlex quasi-static conditions

Design Matrix						Responses					
T1D1 [N]	T1D2 [N]	T2D1 [N]	T2D2 [N]	T3D1 [N]	T3D2 [N]	x [µm]	y [µm]	z [µm]	θ_x [µrad]	θ_y [µrad]	θ_z [µrad]
0,6	-0,2	-1	-1	1	-0,2	-23	0	-403,37	15723,33	27294,4	-8057,19
-0,6	-0,6	0,2	1	-1	0,2	0	-39,85	172,88	-47310,09	-27294,38	5583,32
-0,2	0,2	-0,2	-1	-1	0,6	11,5	-59,78	-57,63	15769,99	54588,86	3098,05
-1	-0,2	1	0,6	-0,2	1	-23,01	39,85	403,37	-39390,09	13647,29	6831,66
1	0,2	1	1	0,6	0,2	69,02	79,71	403,37	-15723,33	-27294,4	-1883,93
-1	-1	-0,2	-1	-0,2	-1	-57,52	-19,93	-864,35	-116,71	-0,06	3120,86
-0,6	0,2	-1	1	0,2	0,6	-69,02	-39,85	518,61	-23596,69	-13647,16	-1838,28
0,6	-0,6	0,6	0,6	0,2	-1	46,01	39,85	-288,11	-15816,67	-54588,89	-635,59
-1	1	-0,6	-0,2	-0,6	0,2	-57,52	-59,78	288,1	39483,43	13647,2	3120,87
0,2	1	0,2	1	-0,2	-0,6	23,01	0	403,35	31610,14	-54588,88	612,76
-0,6	0,6	0,2	-1	0,2	1	-34,51	19,93	172,86	23690,01	68236,09	1872,51
1	-1	-0,2	0,6	-0,6	-0,2	69,02	-39,85	-172,86	-47356,77	-27294,41	-1883,92
-1	0,6	-0,2	0,6	0,6	-0,2	-80,53	19,93	288,11	15816,74	-27294,44	646,99
-1	0,2	0,6	-0,6	1	-0,6	-69,02	79,71	-288,12	31516,73	-0,05	1883,92
1	1	0,6	-1	-0,2	0,2	80,52	19,93	57,61	55230,1	40941,61	-646,99
1	0,6	-1	0,2	0,2	-0,6	23,01	-39,85	57,61	31563,44	-27294,47	-6831,66
0,2	1	-0,6	-0,6	0,6	-1	-23,01	0	-172,89	70976,84	-13647,3	-4334,97
0,6	0,6	-0,6	0,6	-1	1	46,02	-79,71	633,86	-7803,34	13647,27	-635,57
0,2	0,2	0,6	0,2	-0,6	-0,2	46,01	0	57,62	7896,7	-13647,22	3086,63
0,6	-0,2	0,2	-0,6	-1	-0,6	69,02	-39,85	-403,37	15723,35	-0,04	1838,29
0,6	-0,6	1	-0,6	-0,6	0,6	80,52	19,93	-172,86	-23690,1	40941,67	3075,22
0,2	-1	0,2	0,2	1	1	-11,5	59,78	57,64	-63103,51	27294,51	-3098,05
-0,2	0,6	1	-0,2	-1	-1	46,01	0	-172,88	47310,13	-27294,5	6808,84
-0,6	-1	1	-0,2	0,6	-0,2	-23,01	79,71	-403,35	-31610,1	0	3109,44
-0,2	-0,6	-0,6	-0,6	0,2	0,2	-34,51	-19,93	-288,11	-15816,74	27294,44	-1849,69
-0,2	-0,6	-0,6	1	1	-0,6	-57,52	19,93	-57,62	-31563,37	-54588,85	-4323,57
-0,6	-0,2	-1	0,2	-0,6	-1	-46,01	-79,71	-288,12	7850,04	-40941,69	635,59
-0,2	1	0,6	0,2	1	0,6	-23,01	79,71	518,6	23736,74	13647,23	-612,77
1	-0,2	-0,2	-0,2	0,6	1	34,51	19,93	172,88	-23643,42	40941,7	-5594,73
0,2	-1	-1	-0,2	-0,2	0,6	-11,5	-59,78	-172,85	-47356,81	27294,48	-3098,03

Table 4: Uniform Design and results of the Experiments

$$
\begin{aligned}
x &= \beta_{1x} + \beta_{2x}\mathrm{T1D1} + \beta_{3x}\mathrm{T1D2} + \beta_{4x}\mathrm{T2D1} + \ldots \\
&\quad \beta_{64x}\mathrm{T1D1}\cdot\mathrm{T1D2}\cdot\mathrm{T2D1}\cdot\mathrm{T2D2}\cdot\mathrm{T3D1}\cdot\mathrm{T3D2} \\
y &= \beta_{1y} + \beta_{2y}\mathrm{T1D1} + \beta_{3y}\mathrm{T1D2} + \beta_{4y}\mathrm{T2D1} + \ldots \\
&\quad \beta_{64y}\mathrm{T1D1}\cdot\mathrm{T1D2}\cdot\mathrm{T2D1}\cdot\mathrm{T2D2}\cdot\mathrm{T3D1}\cdot\mathrm{T3D2} \\
z &= \beta_{1z} + \beta_{2z}\mathrm{T1D1} + \beta_{3z}\mathrm{T1D2} + \beta_{4z}\mathrm{T2D1} + \ldots \qquad (4) \\
&\quad \beta_{64Z}\mathrm{T1D1}\cdot\mathrm{T1D2}\cdot\mathrm{T2D1}\cdot\mathrm{T2D2}\cdot\mathrm{T3D1}\cdot\mathrm{T3D2} \\
\theta_x &= \beta_{1\theta_x} + \beta_{2\theta_x}\mathrm{T1D1} + \beta_{3\theta_x}\mathrm{T1D2} + \beta_{4\theta_x}\mathrm{T2D1} + \ldots \\
&\quad \beta_{64\theta_x}\mathrm{T1D1}\cdot\mathrm{T1D2}\cdot\mathrm{T2D1}\cdot\mathrm{T2D2}\cdot\mathrm{T3D1}\cdot\mathrm{T3D2} \\
\theta_y &= \beta_{1\theta_y} + \beta_{2\theta_y}\mathrm{T1D1} + \beta_{3\theta_y}\mathrm{T1D2} + \beta_{4\theta_y}\mathrm{T2D1} + \ldots \\
&\quad \beta_{64\theta_y}\mathrm{T1D1}\cdot\mathrm{T1D2}\cdot\mathrm{T2D1}\cdot\mathrm{T2D2}\cdot\mathrm{T3D1}\cdot\mathrm{T3D2} \\
\theta_z &= \beta_{1\theta_z} + \beta_{2\theta_z}\mathrm{T1D1} + \beta_{3\theta_z}\mathrm{T1D2} + \beta_{4\theta_z}\mathrm{T2D1} + \ldots \\
&\quad \beta_{64\theta_z}\mathrm{T1D1}\cdot\mathrm{T1D2}\cdot\mathrm{T2D1}\cdot\mathrm{T2D2}\cdot\mathrm{T3D1}\cdot\mathrm{T3D2}
\end{aligned}
$$

(5)

Using a penalized least squares regression [19, 20] to found each β coefficient for each equation in Eq.4, the model is simplified to the linear system shown in Eq.6.

The matrix S_T in Eq.7 represents the input-output matrix of the mechanism; this matrix is consistent for units of micron (for translations), microradians (for rotations) and Newtons (for input forces in Tabs).

$$\begin{bmatrix} x & y & z & \theta_x & \theta_y & \theta_z \end{bmatrix}^T = S_T \begin{bmatrix} \text{T1D1} & \text{T1D2} & \text{T2D1} & \text{T2D2} & \text{T3D1} & \text{T3D2} \end{bmatrix}^T \tag{6}$$

where,

$$S_T = \begin{vmatrix} 57.53 & 0 & 28.76 & 0 & -28.76 & 0 \\ 0 & 0 & 49.82 & 0 & 49.82 & 0 \\ 0 & 287.66 & 0 & 287.66 & 0 & 287.66 \\ 0 & 39313.03 & 0 & -19679.85 & 0 & -19633.3 \\ 0 & 0 & 0 & -34032.01 & 0 & 34060.19 \\ -3101.84 & 0 & 3101.79 & 0 & -3101.81 & 0 \end{vmatrix} \tag{7}$$

The inverse model can be found directly from Eq. 6.

$$\begin{bmatrix} \text{T1D1} & \text{T1D2} & \text{T2D1} & \text{T2D2} & \text{T3D1} & \text{T3D2} \end{bmatrix}^T = S_T^{-1} \begin{bmatrix} x & y & z & \theta_x & \theta_y & \theta_z \end{bmatrix}^T \tag{8}$$

To validate the accuracy of the the metamodel, 1000 aleatory experiments using inverse model are made to compare the pose estimations using metamodeling against the FEM software Ansys. The precision of the model is calculated using the maximum absolute error ($MAXABS$ Eq.9) and the root mean square error ($RMSE$ Eq.10). The $MAXABS$ allows to calculate the local error and the $RMSE$ provides good estimate of the global error. The error between metamodel predictions and Ansys results is shown in Table 5.

$$MAXABS = max\left\{|y_i - \widehat{y_i}|\right\}_{i=1,\dots,n_{error}} \tag{9}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n_{error}} (y_i - \widehat{y_i})^2}{n_{error}}} \tag{10}$$

	MAXABS	RMSE	%error
x	4,01E-04	8,67E-05	1,08E-03
y	4,18E-04	2,07E-05	3,59E-04
z	2,85E-04	4,19E-05	2,41E-04
θ_x	2,41E-02	9,78E-04	2,26E-03
θ_y	4,16E+00	5,57E-03	6,21E-01
θ_z	2,10E-02	2,23E-03	2,30E-03

Table 5: Error between metamodel estimations and Ansys simulations, for 1000 aleatory experiments

4 Conclusions

This article it presents a computer-based metamodeling methodology for modeling compliant mechanisms under quasi-static conditions using design of computer experiments. The methodology is applied in the analysis of the HexFlex, a six degrees of freedom compliant mechanism. The model found for the HexFlex shows a good accuracy with a max error of 0.621% after making 1000 experiments of the metamodel and compare them with the vitual FEM model. Realizing Factorial Designs it was possible to identify characteristics of the behavior of the mechanism; as the precence of symetries in the actuation and the quasi-static behavior of the mechanism. To finaly model the mechanism an Uniform Design of experiments was adopted. The mechanism was modeled using tools for Response Surface Metodology and a low-order polynomial, as a consecunece of its quasi-static behavior. The founded model allows to have an input/output model of the mechanism giving a transfer equation for developing model-based control, and giving tools to the design for optimal implementation of the mechanism in different applications.

An advantage of the method is that it is possible to found a model based control of a compliant mechanisms juts with computer experiments, reducing costs of experimentation and product development.

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