TOPOLOGICALLY CONSISTENT PARTIAL SURFACE RECONSTRUCTION FROM RANGE PICTURES[1]

OSCAR E. RUIZ S.
Center for Interdisciplinary Research (CII)
EAFIT University
A.A. 3300, Medellin
oruiz@eafit.edu.co
Colombia

PETER NEUGEBAUER
Medical Imaging and Cognitive Computing Division
Fraunhofer Institute for Computer Graphics
6 Rundeturmstr, Darmstadt D-64283
neugeb@igd.fhg.de
Germany

ABSTRACT

Surface or shape reconstruction from 3D digitizations, range pictures play an important role as the sizes and accessibility of the sampled object become intractable. Range pictures, however, present challenges regarding: (i) recovery of topological structure from geometrical information of a partial view: (ii) inclusion of several or self obstructing objects on the same picture, and (iii) conciliation of partial topological and geometrical information from the individual views into a main model. Issue (i) and (ii) require introduction of data structures and algorithms able to consistently represent incompleteness and discontinuities in the surface. Aspect (iii) demands the application of statistical methods to sort redundant e inconsistent information in the overlaps between the individual views. In this investigation, tasks (i) and (ii) have been undertaken by designing and populating an extended Boundary Representation (B-rep), using equivalence relations that induce partitions on the data sets. Task (iii) has been carried out by using data processing tools (DigitLAB) that filter, resample, and recover shape from planar digitizations, by applying formalisms of differential topology.

keywords: shape reconstruction, pattern digitization, partial meshes, range images.

1. INTRODUCTION

Range picture sampling allows to take partial views of the removed object. Based on camera calibration and picture registers, partial point samples are available for the shape reconstruction algorithms. Frequently, a large number of pictures must be taken in order to sample the whole external surface of an object, with an acceptable degree of quality of the data. The quality, in this context, refers to the completeness of coverage, the guarantee of a surface-sampling interval adequate to the object details and the accuracy of the point data gathered.

Considerable progress has been achieved in reconstructing solid information from the range pictures by using, for example, the Marching Cubes algorithm and integration of its output. These efforts include the picture integration, and the application of heuristics for guessing parts of the object that have not been covered by the pictures, or cannot be effectively separated from the background, or are represented by samples with large error factors. For geometric modeling from range pictures, the usual Boundary Representation (B-Rep), a watertight model with complete surface, presents some difficulties in: (i) the correct guessing of the hidden parts of the object, and (ii) the conciliation of information between overlapping pictures. Regarding (i), it is shown here that there are many cases in which it would be more convenient to have incomplete (although consistent) shell representations, corresponding to the partial pictures. This feature enables treatment of masks (incomplete B-reps with zero thickness). Regarding (ii), mesh zippering algorithms have been proposed, but evidence presented in technical forums show that topological and geometrical problems must still be addressed and solved. Specifically, catastrophic results are obtained with low density data, that means, sampling objects whose features have high (spatial) frequency contents.

This article reports results in construction of partial masks from range pictures, and their integration to obtain either watertight complete models, or larger incomplete shells. The fitting of partial, topologically consistent surfaces to point data set requires the definition of formalisms and data structures, similar to B-rep ones, but which explicitly record the <u>absence</u> of surface in certain neighborhoods. The issue of conciliation of overlapping pictures is attacked here by applying tools developed in DigitLAB, which allow to generate virtual digitizations from physical ones through classification, re - sampling, filtering, etc. The application of these tools to several data sets has shown that preparation of the data significantly improves the results of geometric algorithms. Because the data deficiency

[1] Proceedings of the Computer Graphics and Imaging (CGIM) conference of IASTED (Intl. Assoc. of Science and Technology for Development). Las Vegas, USA. Nov. 19-23, 2000.

inherent to the problem of shape reconstruction all algorithms fail when applied to raw samples.

In this article, section 2 presents a literature review of topics involved. Section 3 discusses the algorithms and data structures used. Section 4 displays the results of application of the proposed methodology and concludes the paper.

2. LITERATURE REVIEW

The main topics for surface reconstruction from point sets are data capture, topology recovery (the process to identify and formally represent neighborhood information in the data set), and continuity enforcement (smoothing). The interested audience is invited to read [1] for deeper insight on general topics of reverse engineering.

Data Acquisition: Acquisition may be by contact or remotely. Contact measurement is based on the position of the kinematic joints holding a probe that touches the object. In most metrology centers (Coord. Measurement Systems - CMS) planar sampling trajectories have 3 degrees of freedom (X-Y-Z table), and therefore recondite features are not reachable. When more degrees of freedom (dof) are present (with articulated arms), the probe is able to reach creases and holes, at the penalty of manual measurement. Still, approximate planar samples can be obtained pre-processing ([2, 3]) if other characteristics of the digitization (density, homogeneity, etc) are sufficient.

Range imaging records a depth field in grid patterns corresponding to pixel arrays. Each pixel has associated the coordinates (x,y,z) of the surface point hit by the ray passing through the pixel, as well as the vector describing the ray. The grid data so obtained contains implicit neighborhood information that facilitates topology reconstruction.

Topology Recovery: In the surveyed literature Alpha Shapes ([4]) and Marching Cubes ([5]) are used as engines for recovering topology information ([6,7]). In this article an alternative scheme will be followed. B-Rep models ([8]) do not directly serve surface reconstruction since they are watertight closed. Therefore, an extended B-Rep structure is devised here to record absence of surface and existence of borders on some parts of the recognized surface or partial mask (possibly with holes). Since authors [1, 11] report the difficulty in completing or inferring lost or hidden regions of the surface, the present article assumes that such regions should not be inferred. Rather, algorithms should only recover the portion of the object actually witnessed, leaving to different geometric reasoning the artificial completion of surface portions. Hollow, partial objects with holes (for example a carnival mask) are cases in which no completion should be made. Therefore, inferring un-sampled portions of the object is beyond the scope of this article. Regarding the carrier geometries of the shells, this investigation uses very simple geometries such as 3 and 4 - vertex facets. The last ones are of course not flat in general, but are easily subdivided into triangles. These primitives have been found sufficient to support a correct topology.

In efforts for conciliation of meshes from range images Turk & Levoy [9] use a user provided – alignment to snap a image into another by finding a rigid transformation that, applied to one image minimizes the distance with the other. In [10], Curless & Levoy demonstrate however that [9] fails for cases of high curvature objects. Both approaches intend to build a closed shell, but while [9] erodes overlapping portions of the shells, [10] creates the shells from implicit surface in R^3 ($\boldsymbol{f(p) = 0}$), defined by a statistical reliability associated to portions of the pictures.

Surface Smoothing: Once a topologically correct shell is attained, applications may require a level of continuity (typically $\boldsymbol{C}^1$ or $\boldsymbol{C}^2$) on the surface built. Publications [6,7] start with a topologically correct $\boldsymbol{C}^0$ shell, and cover it with vertex, edge and face charts in order to obtain a complete mapping between the $\boldsymbol{C}^0$ shell and a manifold M. This mapping enables the definition of a chart-depending parameterization that produces a $\boldsymbol{C}^2$ continuous surface by using generalized B-spline surfaces.

Mentioned in the literature [1,7], and from our own experience, it appears that a considerable effort may be spent in ensuring $\boldsymbol{C}^1$ or $\boldsymbol{C}^2$ continuity in selected regions of the object where only $\boldsymbol{C}^0$ continuity exists. Sharp "character" edges are present in objects (for example car bodies), and their detection already presents formidable difficulties. Therefore, for many applications, the smoothing of the surface is not yet a pressing issue.

From the survey presented, it is clear that industrial usage of digitization tools requires foremost topological correctness. In dealing with range pictures, partial, topologically consistent shells must be produced as a first step. Patterned samples (grid or planar) present very attractive characteristics for topology recovery. Once it is achieved, picture information conciliation proceeds. This article discusses tools that allow to retrieve such correct shells by profiting from sample patterns and to process them to prepare the data for the conciliation of range picture images. This last stage will be achieved by application of the surfacing algorithms from DigitLAB ([2,3]). The approach used here will be to avoid deletion of any overlapping point data. Rather, these portions are averaged by planar filtering, and all points there will be used to generate planar cross cuts of the object. No minimization is used, therefore accelerating the algorithm.

3. METHODOLOGY

This section addresses the issues of (i) topological and geometrical algorithms to use range pictures in recovering incomplete shells from range pictures, and (ii) statistical and geometrical tools applied to attack the problem of conciliation of shells originated in overlapping range pictures.

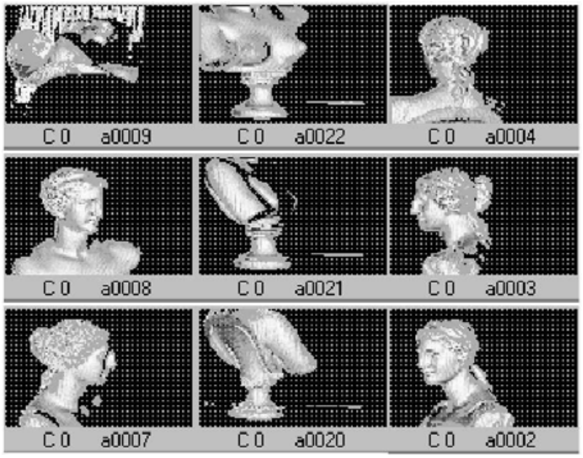

FIGURE 1. Aphrodite data set as processed by registration software (SCULPTOR, Fraunhofer IGD).

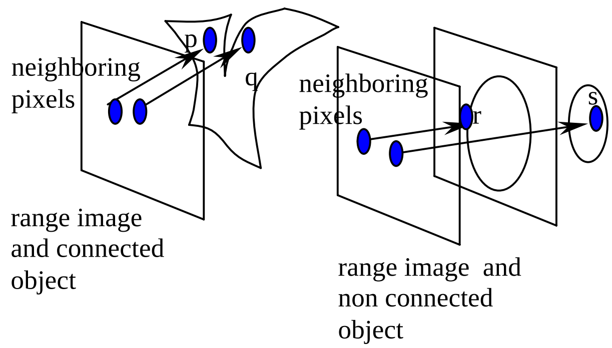

FIGURE 2. Variation in the topological location of neighboring pixels in the range image.

3.1 Partial Shell Construction

Figure 1 shows a series of calibrated and registered range pictures in the SCULPTOR software (Fraunhofer Institute for Computer Graphics). Each pixel ***(u,v)*** in a picture contains the information of the ***(x,y,z)*** coordinates of the object surface that are touched by the view ray passing by the pixel (along with other data not discussed here). SCULPTOR makes the necessary calculations to present the data as would be produced by parallel rays impacting the object. From the calibration process, the black areas are considered either as background or non captured by the picture, and therefore the range picture presents no evidence of object existence in the corresponding pixels ***(u,v)***.

The immediate goal when having range pictures is to recover neighboring information present in them. However, care must be exercised since neighboring pixels in the range image (Figure 2) may correspond to points that are very far away on one surface or are points on different surfaces or different objects. This consideration leads to two conclusions: (i) The topological data chosen must be flexible enough to express separate shells on each picture, each one incomplete and possibly with holes. (ii) The algorithms to recognize and extract those shells must account for propagated neighborhoods based on a transitive proximity relation, rather than a direct, simple Euclidean criteria. Based on this concept, eventually points ***p*** and ***q*** enter the same shell (left case), while points ***r*** and ***s*** will never share a shell.

3.2 Spatial Relations for Shell Building

The definition of the basic relations between points are discussed next. Let (u_0,v_0), (u_1,v_1), and (u_2,v_2) be three pixel vertices, representing p_0, p_1, p_2 points on the object surface. Let us define:

image_unit_triangle: three pixel vertices (u_0,v_0), (u_1,v_1), (u_2,v_2) form a ***image_unit_triangle*** if :$|u_i - u_j| \leq 1$ and $|v_i - v_j| \leq 1$ (they are immediate neighbors in the grid)

object_triangle: three points p_0, p_1, p_2 form a triangle on the object surface (***object_triangle***) if
(i) ***image_unit_triangle***$((u_0,v_0), (u_1,v_1), (u_2,v_2))$ and
(ii) $| p_i - p_j | \leq \delta_t$, $(0 \leq i , j \leq 2)$.

This means, the three vertices are immediate neighbors on the grid and represent samples closer than δ_t on the object.

transitive_neighbors: two points p, q on the object are ***transitive_neighbors*** if $\exists$ $path(p,q)=[\ p=p_0,\ p_1,\ldots\ p_n=q\ ]$ sequence of points p_i such that ***object_triangle***$(p_i ,\ p_{i+1} ,\_\_)$ $(0 \leq i \leq n\text{-}1)$. Therefore, $| p_i - p_{i+1} | \leq \delta_t$, $(0 \leq i \leq n\text{-}1)$. This means, they are part of a chain of neighboring triangles, and $path(p,q)$ is a path with traversal steps no larger than δ_t. Informally, one may say that there is a trail of object triangles that contains ***q*** and ***p***.

Notice that ***transitive_neighbors()*** is an equivalence relation (it is symmetric, reflexive, transitive). Therefore it induces a partition on the point set ***S***. The sets of the partition are exactly the separate shells S_i registered in the range picture.

$$S = \cup S_i \ ,(0 \leq i \leq Ns) , \qquad S_i \cap S_j = \phi , \qquad i \neq j \quad (1)$$

No two shells may share a vertex (otherwise they would be one). Surface points not belonging to any triangle are not considered here. For the sake of mathematical formality, it would be appropriate to consider them as making up a triangle with themselves. From the engineering point of view, however, they may be purged from the data set. Notice that the definition above allows incomplete shells with holes, as required.

3.3 Algorithms for Shell Building

The algorithm for partitioning the set S according to the ***transitive_neighbors()*** relation is basically one of building the closure of sets (the shells) under the relation. Each shell is started by a seed triangle and grows as triangles adjacent to the shell boundary satisfy the ***object_triangle()*** relation. The relation ***image_unit_triangle()*** is necessary (but not sufficient) for ***object_triangle()***. Therefore, the search for promising triangles starts on the ***(u,v)*** pixel space (where pixel neighborhood is easily tested) and completed in the object space to satisfy the metric constraints. In general, the shell boundary includes external and internal contours(see Figure 4.b)

The simplified algorithm for shell expansion is displayed in Figure 3. A shell grows from a seed triangle, by using the edges in the boundary B as candidates to absorb new triangles satisfying the relation ***object_triangle()*** (line 6). If no growth is possible, it means the boundary is stable and the current shell on the surface of the object, as registered by the image has been completely represented (line 8). If a triangle next to the boundary satisfies ***object_triangle()*** , it must be incorporated to the current shell SH (line 11) and accounted for in the current boundary B (line 12). It must be noticed that shell growth is marked in the macro algorithm as "**SH**=**SH**+[**T_new**]". Similar notation is applied for boundary growth.

```
shell SH] =shell_expansion(     triangle seed ,
                                image_grid G )
{
1  SH = make_shell( seed );
2  B = extract_boundary( seed ) ;
3  growing  = TRUE;
4  while (growing)
5  {
6       T_new = grow_candidate( B, G, SH)
7       if (not T_new )
8            growing = FALSE;
9       else
10      {
11           SH=SH+[T_new]
12           B = B + [boundary(T_new)];
13      }
14 }
15 return( SH );
}
```

FIGURE 3. Macro-algorithm for shell expansion.

3.4 Shell and Boundary Growth

Shell and boundary expansion are present when an edge ***e*** in the current boundary and a vertex ***v*** in the immediate neighborhood represent a new triangle that satisfies the relation ***object_triangle().*** The vertex must be in condition of allowing yet one more face incident on it (it cannot be part of the interior of the mesh). Figure 4 displays the possible cases, showing the new unit triangle in a darker tone. Based on their topological effects they are: general expansion (4.a), corner fill (4.b), hole fill (4.d) and contour splitting (4.c). Two different meshes never share a vertex. Boundary self-intersection (4.e) can be avoided by using the fact that the ***(u,v)*** space is a discrete one, and therefore there can be no vertex of a triangle inside a unit lattice in the ***(u,v)*** grid. Every vertex in the *interior* of the shell has closed incidence and therefore it cannot receive any other incident edge (4.f). Conversely, ***v*** might be a vertex *on* the current boundary and therefore it would have its incidence degree still open (4.c). The relation ***object_triangle()*** evaluates whether ***v*** and edge ***e*** form a plausible triangle depending on its geometrical quality. Shell growth by "general expansion" (4.a) implies that the triangle being included in the mesh adds two new expandable edges to the boundary. A corner fill implies a reduction of the expanding boundary while keeping topology unchanged. Otherwise, mesh boundaries

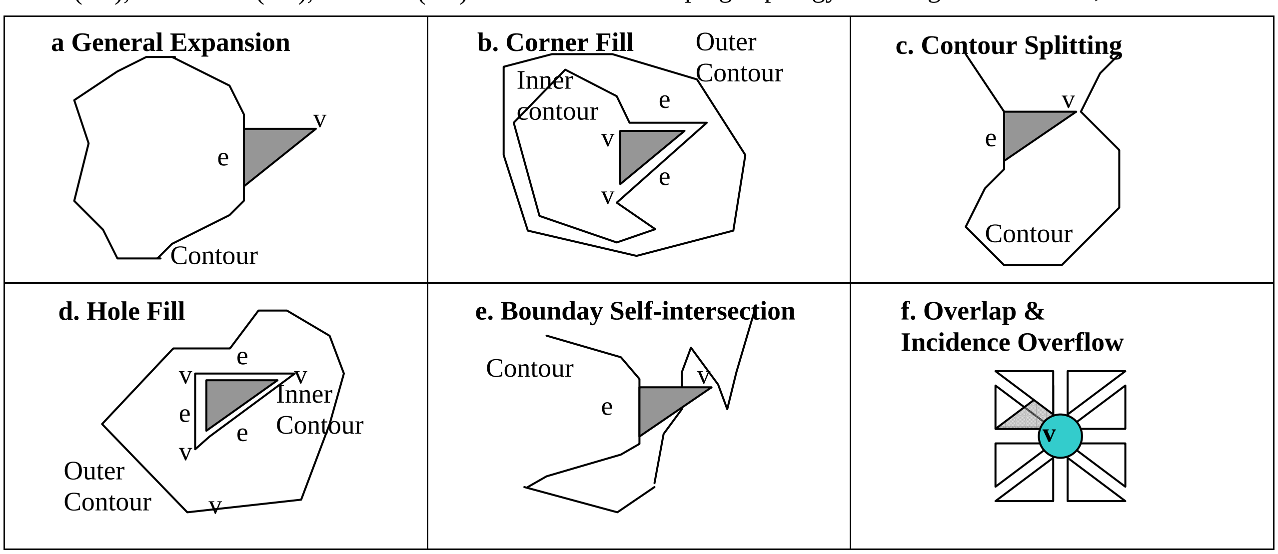

FIGURE 4. Topological – geometrical cases for expansion of shell boundary.

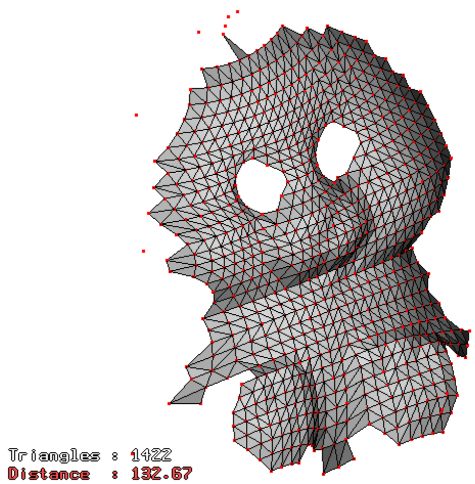

FIGURE 5. “Teddy Bear” data set. Shell recovered from one range picture.

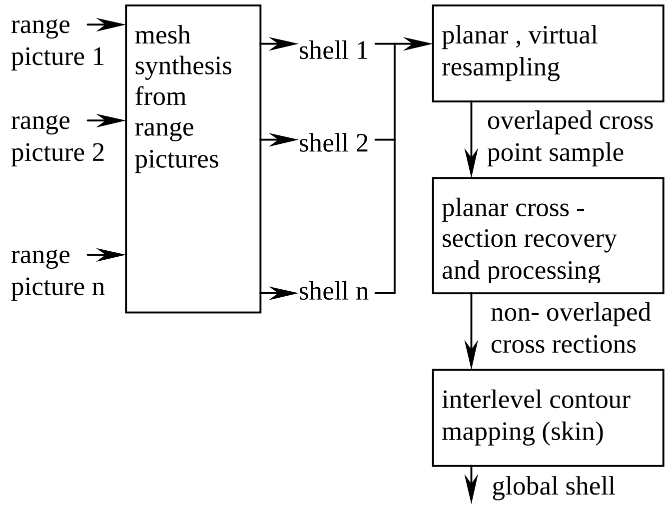

FIGURE 6. Integration of shells from different range pictures

may be are born (4.c) or collapsed (4.d).

3.5 Representation of Incomplete Shells

Partial shells, very common in engineering applications, require an extended Boundary Representation to explicitly account for the missing parts of the shell. In normal B-reps FACEs may have holes, and are bounded by LOOPs, one of which is the outermost and the others bound internal holes. Each EDGE appears exactly two times in the B-Rep, traversed in two opposite directions, according to the two (neighboring) FACEs that own it. In partial B-reps from range pictures: (a) there are several incomplete shells, coming from one or several object(s), (b) FACEs are triangles. There are no internal LOOPs, every FACE has exactly tree EDGEs, it is flat, and admits no holes, and (c) EDGEs in the boundary of the SHELL (or mesh) are by definition the ones that have only one incident FACE.

The discussion above suggests that an explicit recording of the contours that limit a shell is required, both to drive the shell growth and to register holes in it. Figure 5 shows the shell recovery results for the “Teddy Bear” data set, which corresponds to a hollow physical mask. This result shows the possibility to accommodate holes in the emerging shell. The algorithm ignores isolated points since they cannot be integrated to the shell, and they do not pass to the next stage of the treatment (conciliation of several range pictures).

3.6 Conciliation of Overlapping Images

The conciliation of shells or meshes originated in overlapping images is in general a difficult and expensive process. As seen in Figure 5, triangles in the periphery of the shell are sharper than triangles which stand in front of the incident rays and present a better shape ratio and statistical certainty([9,10]). In order to deal with the overlapping zones in a statistical way, the process shown in Figure 6 was followed in DigitLAB: (i) Partial Shells are extracted from range pictures, by using the growing mesh algorithm discussed above. (ii) One integrated data set is obtained by planarly re-sampling the set of partial meshes as if the object had been sampled on a XYZ metrology table (Coordinate Measuring Machine, CMM). This data set still contains fuzzy, oversampled regions, originated in the shell overlaps. (iii) Planar cross cuts are recovered, although they present noise in the overlapping regions. Statistical and filtering processing is applied on them. (iv) The skin is calculated between contours in continuous levels, producing a global shell formed by quadrangular or triangular facets. The process is also tuned to produce good quality (aspect ratio) facets useful in Finite Elements applications. The topologies considered are basically Morse-Simple at this stage. There are not degenerate critical points. For a deeper review about Morse Theory see ([12]). For DigitLAB see [2, 3].

4. RESULTS AND CONCLUSIONS

Figures 7 and 8 show two stages of the treatment in DigitLAB for the Aphrodite data set. Figure 7 presents two views of a partial shell. Figure 8, left, is the superposition of several partial shells extracted from the range pictures. There exist missing parts wherever the pictures had a dark spot or in places covered by no image. Figure 8, right, corresponds to the integration of all separate shells. The integration was carried out by applying a planar re-sampling on all separate shells, cross section recovery, filtering and re-sampling of the cross sections, and finally, inter-level maping or lofting. The missing parts where completed by point projection between levels. The heuristics for incomplete shape guessing require more investigation, since they produce unstable results depending on the situation handled. This article seeks to concentrate only on the individual shell recovery and conciliation, and therefore this issue will be addressed in other publications. The authors currently work on the application of topological considerations in order to guarantee the correctness of the shells or masks build from both planar cross cuts and range

FIGURE 7. “Aphrodite” data set. Two views of a partial shell from a range picture.

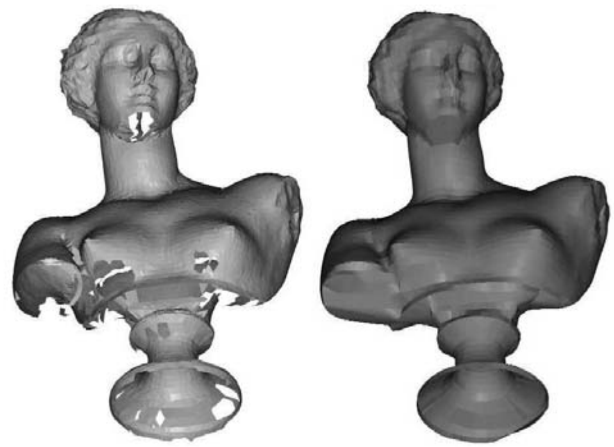

FIGURE 8. “Aphrodite” data set. Conciliation of shells from range pictures.

images. It is evident that this is an issue requisite to any other (for example, smoothness).

Pattern samples are common in medical and industrial environments (magnetic resonance, tomographies, metrology, etc), while grid ones are present in ergonomics, cloth design, entertainment, computer vision, etc. Therefore, the authors believe that the material presented here is relevant for the topological correctness of the shape recovered and therefore impacts all downstream applications.

Acknowledgments. The authors wish to thank the economic support of the Deutscher Akademischer Austaushdienst (DAAD) for the internship of Assoc. Prof. Dr. Oscar Ruiz University at Fraunhofer IGD’s in 1999 and the Colombian Institute for Science and Technology (Colciencias) through grant FALCON-98 and 99.

REFERENCES

[1] Varady, T., Reverse Engineering of Geometric Models – An Introduction. *Computer Aided Design*, Vol 29, No 4, 1997, pp 255-268.

[2] Ruiz, O., Posada, J., Computational Geometry in the Preprocessing of Point Clouds for Surface Modeling. In Book *Integrated Design and Manufacturing in Mechanical Engineering-98*. ISBN 0-7923-6024-9, (Dordrecht, Nederlands, Kluwer Academic Publishers, 1999).

[3] Ruiz, O., DigitLAB, an Environment and Language for Manipulation of 3D Digitizations. *Proceedings of the Joint Conference : IDMME'2000 & CSME FORUM 2000*. Montreal, Canada. May 16-19, 2000, pp 180-199.

[4] Edelsbrunner, H., Three Dimensional Alpha Shapes, *ACM Transactions on Graphics*, Vol 13, No 1, Jan , 1994, pp 43-72.

[5] Lorensen, W., Cline, H., Marching Cubes: A High Resolution 3D Surface Construction Algorithm, *ACM Computer Graphics*, Vol. 21, No. 24, July, 1987, pp. 163-169.

[6] Grimm, C., Hughes, J., Modeling Surfaces of Arbitrary Topology using Manifolds. *Proceedings, SIGGRAPH - 95 Annual Conference Series*, Los Angeles, August 6-11, 1995, pp. 359-368.

[7] Guo, B., Surface Reconstruction: From Points to Splines. *Computer Aided Design*, Vol 29, No 4, 1997, pp 269-277.

[8] Mantÿla, M., *An Introduction to Solid Modeling*, (Rockville, USA, Computer Science Press. 1988).

[9] Turk, G., Levoy, M., Zippered Polygon Meshed from Range Images. P*roceedings SIGGRAPH-94*, 1994, pp 311-318.

[10] Curless, B., Levoy, M., A volumetric Method for Building Complex Models from Range Images. *Proceedings SIGGRAPH-96*, 1996, pp 303-312.

[11] Neugebauer P., Reconstruction of Real World Objects via Simultaneous Registration and Robust Combination of Multiple Range Images. *International Journal of Shape Modeling*, Vol. 3, No. 1&2, 1997, pp 71-90.

[12] Fomenko, A., Kunii, T. *Topological Modeling for Visualization*, (Tokio, Springer Verlag, 1997).