

Manifold Learning with Orthogonal Geodesic Grids

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Abstract In Reverse Engineering, it is capital to find a parametric trimmed surface which approximates a triangular mesh (2-manifold with border) $M \subset R^3$. This article proposes and implements a quasi isometry $f : M \rightarrow R^2$ which allows a parameterization of M . We consider quasi - developable 2-manifolds $M \subset R^3$. $f(p) = (u, w)$ with (u, w) being the coordinates of $p \in M$ under a grid of geodesic curves $C_i(u)$ and $C_j(w)$ on M . We seek that the geodesic curves $C_i(u)$ and $C_j(w)$ be orthogonal to each other on M . This means, that the $C_i(u)$ should not cross each other, and each $C_i(u)$ should intersect each $C_j(w)$ in perpendicular manner.

Keywords manifold learning · surface reconstruction · developable surfaces

1 Introduction

In Reverse Engineering and CAGD, finding a parametric surface in R^3 that approximates a set of points S sampled on a smooth 2-manifold M in R^3 is a central open problem ([5],[2]). The existing approach of Manifold Learning (ML) receives a point sample in a 2-manifold $M \subset R^3$ and maps it to R^2 via a quasi-isometry $f : M \rightarrow R^2$, whenever possible. ML builds a graph whose vertices are the points of S and its edges are straight segments, lying on the 2-manifold M . ML approaches the geodesic path between 2 points on M as the shortest path within this graph. ML seeks to find a set of points S_2 in R^2 ($S_2 = f(S)$), such that the euclidean distance

between two vertices of S_2 is the same as the geodesic distance (in M) between the corresponding vertices of S . This article proposes and implements an alternative $f : M \rightarrow R^2$ as follows: $f(p) = (u, w)$ with (u, w) being the coordinates of $p \in M$ under a grid of geodesic curves $C_i(u)$ and $C_j(w)$ on M (Fig. 1). We seek that the geodesic curves $C_i(u)$ and $C_j(w)$ be orthogonal to each other on M , and the curves $C_i(u)$ be parallel to each other on M (and likewise for the $C_j(w)$). This means, that the $C_i(u)$ should not cross each other, and each $C_i(u)$ should intersect each $C_j(w)$ in perpendicular manner. M being developable is a precondition for this ideal situation. However, we aim to quasi - developable M . This expectation (also present in ML) is a reasonable one in Reverse Engineering by applying Manifold Segmentation. By using this mapping, a family of (u, w) parameterizations of M is reachable, which allows us to develop a trimmed surface representing M (an important landmark in Reverse Engineering).

2 Literature review

In general, a surface is developable if for each point in the surface, its Gaussian curvature is zero [7]. With this in mind, some researches [8,9] in literature focused on approximating a model using developable surfaces. However, these approaches can only model surface patches with 4-sided boundaries as the surfaces are usually defined on a squared parametric domain [2]. Other tries include reconstructing the surface from geodesic interpolation, however, this approach depends strongly on the align of the geodesics. Other methods instead, approximate the surface triangulation with one or more piece-wise polynomial patches to each triangular facet. These methods are weak for large number of triangles, since they generate a large amount of data that it turns to be not practical [6].

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3 Methodology

Fig. 1 presents the origin (red dot) of the geodesic grid to build on M . There are R^2 DOFs to choose this origin. A vector tangent to the containing triangle gives the initial direction of geodesic $C_1(u)$ (θ). Another vector tangent to it, but perpendicular to it, initials the geodesic $C_1(w)$. The direction of this vector pair has R DOFs. The grid separation is an additional DOF. Therefore, in generating a grid we have R^4 DOFs. However, only 2 DOFs are important (grid separation and orientation of angle of initial frame on M). It is easy to find out the evolution of (iso - spaced) $C_i(u)$ and $C_j(w)$ over the triangular mesh. Once all the geodesics are built on the surface, we look for the intersections of the $C_i(u)$ vs. $C_j(w)$ ones, rendering the blue nodes in Fig. 1.

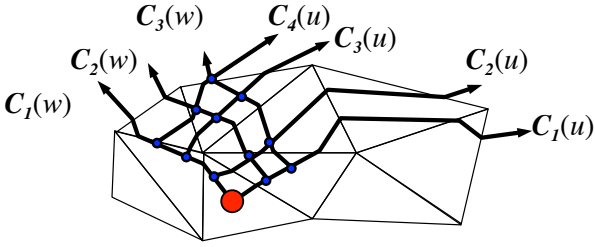


Fig. 1 Origin of Geodesic Grid.

Fig. 2(a) shows the $C_i(u)$ (blue) and $C_j(w)$ (red) geodesics in a developable surface (e.g. cone).

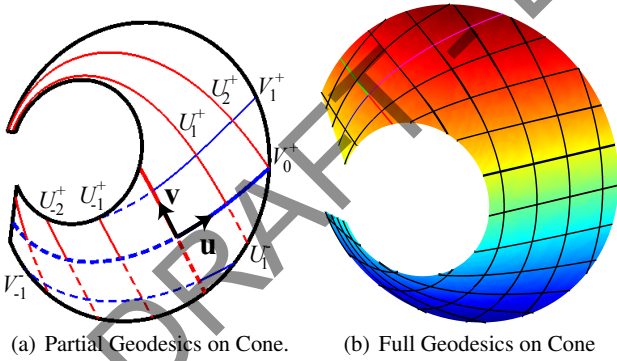


Fig. 2 Grids on developable (cone) manifold.

For the angle θ that maximizes the number of intersection points on the grid we extend the grid of geodesics until all the manifold is covered. Once all the manifold is covered, we deal with the intersections of 'parallel' geodesics $C_i(u)$ vs. $C_j(u)$ (Fig. 3(a)). Every time a intersection is found, the polylines corresponding to the geodesics in the manifold are inter-changed after the intersection point (Fig. 3(b)).

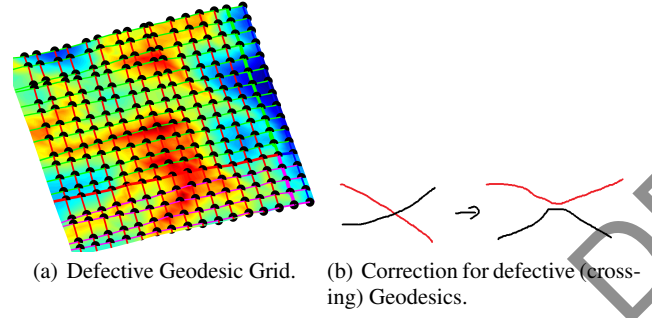


Fig. 3 Defective geodesic grid on non-developable M .

4 Results

Fig. 3 shows early and finished status of the geodesic grid for a developable surface (e.g. cone). In this case, obviously, the fundamental assumption of orthogonality of the grid mesh is true. Fig. 3(a) shows a defective grid, resulting from a non-developable manifold. In this case, geodesic curves $C_i(u)$ and $C_j(u)$ intersect, therefore challenging the expectation of 'parallelism' of them. In such case, we use a heuristic remedy to force the separation of the $C_i(u)$ and $C_j(u)$ by re-defining them as per Fig. 3(b).

5 Conclusions and Future work

The implemented process must be expanded to synthesize a parametric surface $S(u, w) \in R^3$ that approximates and contains M . This goal requires an iterated application of the discussed process. This is the next step in our research.

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