Microscale Investigations of Highfrequency Wave Propagation Through Highly Porous Media

David Uribe[1,3*], **Erik Saenger**[2], **Ralf Jänicke**[1], **Holger Steeb**[1], and **Oscar Ruiz**[3]

[1] Institute of Mechanics, Ruhr-University Bochum, Germany
[2] Geological Institute, ETH Zürich, Switzerland
[3] Laboratorio de CAD/CAM/CAE, Universidad EAFIT, Medellin, Colombia

Wave propagation in highly porous materials has a well established theoretical background. Still there are parameters which require complex laboratory experimentation in order to find numerical values. This paper presents an effective method to calculate the tortuosity of aluminum foam using numerical simulations. The work flow begins with the acquisition of the foam geometry by means of a micro-CT scanner and further image segmentation and analysis. The elastodynamic wave propagation equation is solved using a velocity-stress rotated staggered finite-difference technique. The effective wave velocities are calculated and using the fluid and, aluminum effective properties, the tortuosity is determined.

1 Introduction

The use of high resolution tomographic imaging has proven to be an adequate method to find effective material properties of complex structures [1]. Modern image analysis algorithms together with large-scale computations has brought the possibility to bring together different length scales to accurately describe material behavior. The sample is an open-cell foam with a porosity of 0.93. At the micro scale, the foam consists of struts that are joint rigidly at their ends. The struts' bulk material parameters (Young's modulus, Poisson's ratio and density) are known from quasi-static experiments. First, a description of the workflow from the image adquisition to the virtual experimental setup is explained. Afterwards some numeric results are presented and compared with physical experiment data.

2 Methodology

2.1 Image analysis

The Aluminium foam is digitized using a Computer Tomographic scanner. The resolution of the voxels is 60 μm. Using image segmentation and surface reconstruction algorithms it was possible to gather valuable geometric information, similar to Jang [3]. From the segmented data, a regular 3-D grid is constructed that closely resembles the physical geometry. The size of the grid is 400 x 400 x 400 points.

An important use of the gathered geometric information is determining the limit in which the highfrequency range from Biot's theory begins. From Steeb [4], the viscous skin depth can be calculated using equation 1. In this equation η^{fR} is the pore fluid viscocity, ρ^{fR} is the effective pore fluid density and f the frequency. If the pore radius is larger than the viscous skin depthd, then the effective coupling mechanism between the fluid and the solid phases is only inertia driven. This fact will become important in section 2.2.

$$d = \sqrt{\frac{2\eta^{fR}}{2\pi f \rho^{fR}}} \tag{1}$$

$$f_{crit} = \frac{\eta^{fR}}{\pi r^2 \rho^{fR}} \tag{2}$$

2.2 Simulation of a pressure wave propagating in a water saturated aluminium foam sample

The numerical experiments are solved with a parallelized algorithm. The elastodynamic wave equation is discretized using a rotated-staggered-grid finite-difference scheme developed by Saenger [5]. The foam sample is virtually immersed in the center of a water tank of size 400 x 400 x 804 gridpoints. A plane pressure wave is created at one end of the tank parallel to the z-direction. All the walls of the tank orthogonal to the z-direction have periodic boundary conditions. In our simulation

* Corresponding author: Email David.Uribe@rub.de, phone +49 234 32 26468, fax +49 234 32 14229

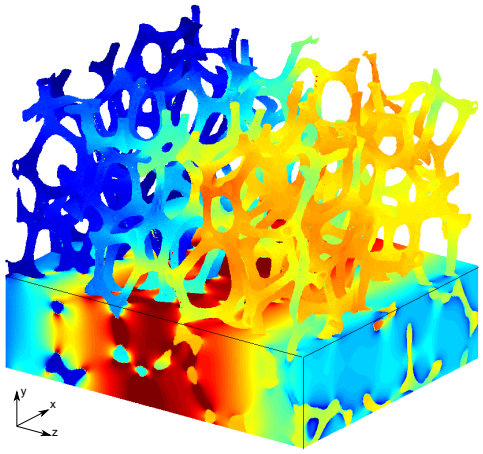

Fig. 1 Aluminium foam with part of the water removed in order to see the structure. Coloring depending on pointwise velocity values.

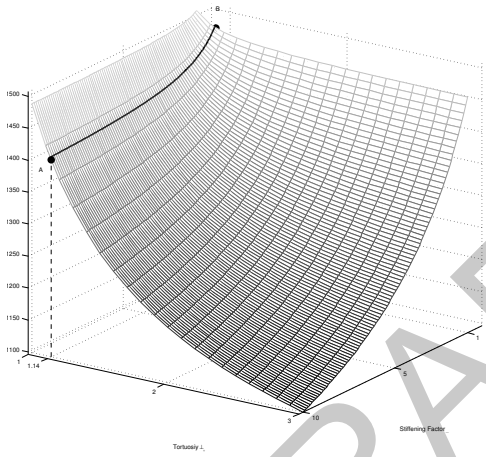

Fig. 2 Analytical solution of the high-frequency limit in Biot's equation using a varying stiffening factor.

we use is a Ricker wavelet, with a central frequency of 1KHz. It can be proven that this frequency is larger than the critical frequency predicted by Biot (eq. 2) and therefore lies in the high-frequency range.

In order to measure the pressure wave velocities, virtual receivers are placed right before the aluminium foam and right afterwards. These measurements are compared with a reference simulation (the water tank with no foam), and then the correct wave velocities can be back calculated.

Artificial aluminium properties

There is the need to create two experiment setups. On one hand, the aluminium Young's modulus and density is multiplied by a factor $\beta = 10$ (point A in figure 2). This increases the impedance contrast between the two phases high enough that the effective wave velocity measured is only influenced by the parameters of the water phase (no coupling between aluminium and water phase). Indeed, it was observed from the simulation that no wave was traveling through the aluminium phase.

From the assumption above, using the high-frequency limit of Biot's equations, the turtuosity can be calculated for the geometry of this foam (since turtuosity is the only unknown). The measured effective wave velocity is $v_{p,fast}$=1409 m/s. This velocity is much lower as the reference velocity of homogeneous water $v_{p,water}$=1480 m/s. The result from Biot's equation for the tortuosity parameter is α_∞=1.14.

Real aluminium properties

Using the real physical aluminium properties in the simulation, there is an effective wave velocity $v_{p,fast}$=1487 m/s. On figure 1 a snapshot in one of the timesteps shows the wave traveling though both phases and through the aluminium phase alone. It can be observed that there is a coherent wave propagating through the water and aluminium phase. On the other hand, there is no coherent (or measureable) wave passing through the aluminium phase by itself.

3 Conclusion

In figure 2, v_p is calculated analytically from Biot's equations for different stiffening values β. If the isoline α_∞=1.14 is plotted over the parametric surface, the value B will be equivalent to calculating the fast P-wave velocity of the water saturated foam with real aluminium properties. Therefore if we compare the values of the numerical simulation and the analytical solution the measure how fit is the method. It was found that the analytical and the numerical solution match exactly. This methodology has proven to be highly effective for the computation of geometric properties such as tortuosity. Our numerical laboratory is not only limited to aluminium foams or homogeneous structures, which makes it a very robust and powerful tool.

References

[1] E. H. Saenger, J. Appl. Phy. **74**, pp. 236–241 (2011).
[2] M. A. Biot, J. Acoust. Soc. Am. **28**, pp. 179–191 (1956).
[3] W. Y. Jang, Int. J. Solids Struct. **45**, pp. 1845–1875 (2008).
[4] H. Steeb, Arch. Appl. 334 Mech. **80**, pp. 489–502 (2010).
[5] E. H. Saenger, Wave Motion **31**, pp. 77–92 (2000).