Estimation of Large Domain Al Foam Permeability by Finite Difference Method.

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The classical methods to calculate permeability of porous media have been proposed mainly for high density (e.g. granular) materials. These methods present shortcomings in high porosity media (e.g. metallic foams). While for dense materials permeability seems to be a function of the bulk properties and occupancy averaged over the volume, for highly porous materials these parameters fail to predict it. Several authors have attacked the problem by solving the Navier-Stokes equations for the pressure and velocity of a liquid flowing through a small domain (Ω_s) of aluminium foam and by comparing them with experimental values (prediction error approx. 9%). In this article we present calculations for much larger domains (Ω_L) using the Finite Difference (FD) method, solving also for the pressure and velocity of a viscous liquid flowing through the *Packed Spheres* scenario. The ratio $Vol(\Omega_L)/Vol(\Omega_s)$ is around 10^3. The comparison of our results with the Packed Spheres example yields a prediction error of 5% for the permeability. We also solved for pressure and velocity in an accurately modelled porous medium. Our geometric modelling of the porous domain stems from 3D X-ray tomography, yielding voxel information, which is particularly appropriate for FD. Ongoing work concerns the reduction in computing times of the FD method, consideration of other materials and fluids, and the enlarging of experimental work.

1 Introduction

Permeability is an important property in the design of filters of metallic foam, porous implants and other applications that require a flow through a porous medium ([1]). In this article we estimate the permeability of high porosity Aluminium foam ($\epsilon > 0.8$) employing the Finite Difference method . For the numerical estimation of the permeability for metallic foams the following methods have been used: (1) Finite Volumes, (2) Lattice Boltzmann, (3) Finite Difference Method. Gerbaux et al. ([2]) calculate the permeability of 3 real metallic foams by solving the Stokes equation in the porous medium with the Lattice Boltzmann method and the finite volume method. Xu et al. ([3]) perform a finite volume analysis to estimate the permeability of some foams cells with different porosities and porous diameter.
Petrasch et al. ([4]) determine porous media properties such as porosity and permeability from a digital representation of reticulate porous ceramics generated by X-ray tomographic scans. Nabovati et al. ([5]) calculate the permeability for fibrous porous media in a wide range of porosity by applying Darcy's law.
At present, only small domains of high porosity metallic foams are addressed for numerical estimation of permeability, given the computing expenses involved. In response to such limitations we implement FD, that is highly compatible with 3D Computer Tomography

2 Methodology

We propose a straightforward method to estimate the permeability of porous media: (1) Discretization of equations 1 and 3 on a staggered grid by using a 2nd order FD method with periodic boundary conditions for velocity in the inlet and outlet of the channel (Fig. 1). (2) Solution of the resulting equation system (Eq. 6, 7, 8, 9) with the method Gauss-Seidel iterative. This method does not require the storage of a coefficient matrix, therefore allowing to simulate larger domains. (3) Calculation of the volume average velocity u_m from the velocity field computed in step 2. (4) Estimation of the medium permeability with Darcy's law (Eqs. 5). The notation used is: $K_D =$ Porous medium permeability according to Darcy's law, $\mu =$ Fluid dynamic viscosity, $u_m =$ Volume average velocity of the fluid in the free volume, $\Delta P =$ Pressure drop in the flow direction.

$$\nabla p = \mu \nabla^2 \mathbf{u} \quad (1) \qquad \nabla \cdot \mathbf{u} = 0 \quad (2) \qquad \nabla \cdot \nabla p = 0 \quad (3)$$

$$u_m = \frac{\int_{V_f} |\mathbf{u}| dV}{V_f} \quad (4) \qquad K_D = \frac{\mu L u_m}{\Delta p} \quad (5)$$

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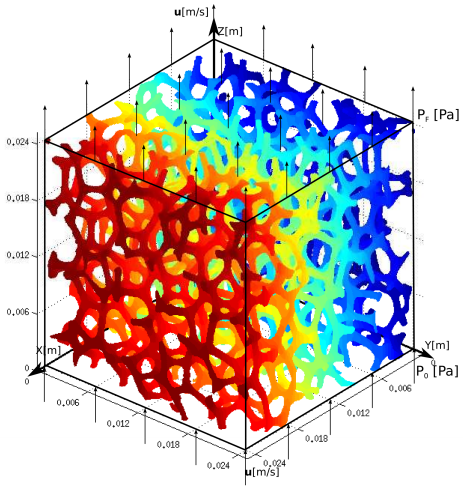

Fig. 1: Domain $400 \times 400 \times 400$ voxels ($24mm \times 24mm \times 24mm$).

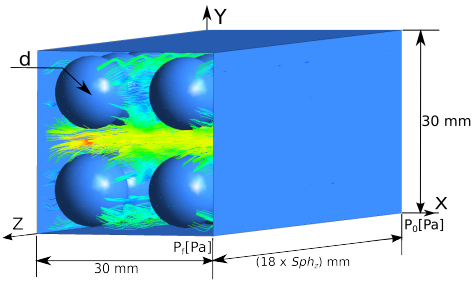

Fig. 2: Regular packed sphere case

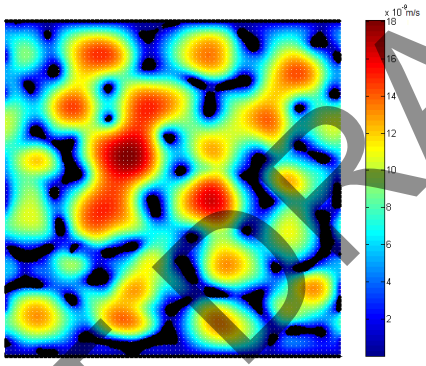

Fig. 3: Velocity in Z direction on plane YZ at x= 12mm.

$$p_{xx} + p_{yy} + p_{zz} = 0 \qquad (6)$$

$$p_x = \mu(u_{xx} + u_{yy} + u_{zz}) \qquad (7)$$

$$p_y = \mu(v_{xx} + v_{yy} + v_{zz}) \qquad (8)$$

$$p_z = \mu(w_{xx} + w_{yy} + w_{zz}) \qquad (9)$$

2.1 Method Validation

The method was validated with a regular packed sphere case in 3D (Fig. 2) whose permeability can be calculated with the Carman-Kozeny model proposed in [6] (Eq. 10) and with the Rumpf and Gupte model proposed in [7] (Eq. 11). The notation used is: K_C = Permeability by Carman-Kozeny model, K_R = Permeability by Rumpf and Gupte model, d = Sphere diameter and ϵ = Medium porosity (Volume free / Volume total). The relative error between the estimated permeability for a regular pack of spheres in a channel with 10 spheres in direction Z ($Sph_z = 10$) and the Rumpf and Gupte model was 5.5% .

$$K_C = \frac{\epsilon^3}{180(1-\epsilon)^2} d^2 \qquad (10)$$

$$K_R = \frac{\epsilon^{5.5}}{5.6} d^2 \qquad (11)$$

3 Results

The permeability of an aluminium foam was estimated for a domain of $24mm \times 24mm \times 24mm$ ($400 \times 400 \times 400$ voxels). Figure 3 shows the velocity in Z direction on plane YZ at x= $12mm$. The calculated results for this case are $u_m = 6.5093 * 10^{-9}\, m/s$ and $K_D = 7.7967 * 10^{-7}\, m^2$. Our implementation allowed the computation of $256 * 10^6$ degrees of freedom in a single processor.

4 Conclusions and Future work

The permeability of a lattice of aluminium foam was calculated with the Darcy's law, using Finite Difference method to simulate a viscous flow through the porous medium. The proposed method optimizes memory usage, therefore allowing to simulate large domains in single processors. Future work includes the reduction in the computing time, the modification of flow parameters such as the input flux and the estimation of the permeability of materials with different porosities.

References

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