COMPUTATIONAL GEOMETRY IN THE PREPROCESSING OF POINT CLOUDS FOR SURFACE MODELING

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ABSTRACT
In Computer Aided Geometric Design (CAGD) the automated fitting of surfaces to massive series of data points presents several difficulties: (i) even the formal definition of the problem is ambiguous because the mathematical characteristics (continuity, for example) of the surface fit are dependent on non-geometric considerations, (ii) the data has an stochastic sampling component that cannot be taken as literal, and, (iii) digitization characteristics, such as sampling interval and directions are not constant, etc. In response, this investigation presents a set of computational tools to reduce, organize and re-sample the data set to fit the surface. The routines have been implemented to be portable across modeling or CAD servers. A case study is presented from the footwear industry, successfully allowing the preparation of a foreign, neutral laser digitization of a last for fitting a B-spline surface to it. Such a result was in the past attainable only by using proprietary software, produced by the same maker of the digitizing hardware.

1. Introduction

In CAD / CAM massive point data sets from digitized objects must be approached by data structures and / or formulae that organize and represent them. This process, called surfacing, is needed for purposes of data reduction, object edition, analysis, visualization and manufacturing. Several problems are present in this endeavor: (i) given a raw set of points, discrimination of functional subsets is needed to preserve the design intent, (ii) given a functionally coherent point set, the mathematical form of the surface fit to it is the designer's choice; (iii) given a mathematical form of the surface, the topological organization of the points is to be decided, and (iv) boundary conditions (C^0, C^1, C^2 continuity) are to be enforced, or waived, depending on functional characteristics of the local neighborhood. Therefore, *automated* fitting of surfaces has proven to be a difficult problem in industry and academy. A traditional solution is to write programs specifically oriented to a family of digitized objects and to lump together digitizing hardware, software and CAD / CAM into one package. This solution

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usually rises the prices of this technology, since the software and hardware only function with each other, thus producing a closed product.

This investigation has aimed to produce and apply a set of open geometric reasoning tools, which take advantage of characteristics of digitizations and data sets to produce surface models. Such characteristics include locking of axis, convex object cross sections or with non empty kernel (Preparata, 1985), oversampling, character lines as curvature discontinuities, etc. To test the tools, a laser digitization was loaded, preprocessed and fit a surface using popular, open CAD systems (Saldarriaga, 1997, Posada, 1997). Originally this could be done only by using vendor's proprietary software operating on data from its own digitizer.

The code for the described tools was implemented to be Application Interface Specification (AIS)-compliant. AIS (Ranyak, 1994, Posada, 1997, Saldarriaga, 1997) describes in functional, generic form the services that a geometric modeler should provide. As a result. the AIS-compatible code written runs transparently on AutoCAD® and MicroStation®.

This article is organized as follows: Section 2 surveys the relevant literature. Section 3 discusses the geometric reasoning tools devised. Section 4 presents a case study while Section 5 concludes the paper. Last section contains the bibliographic references.

2. Literature Review and Background

Surface fitting has been attacked from two different points of view: (i) mathematical and (ii) algorithmic. In (i) mathematical formulae are proposed and fit to the data by parameter identification. Among formulations independent on the order of the data there are interpolations and statistical fittings. Polynomial interpolations present large oscillations and large computational expenses as the set grows. Statistical techniques fit equations which are the most likely to represent the sampled points. These techniques are used for tolerance assessment in which hypotheses of planarity, cylindricity, perpendicularity, etc, are evaluated by sampling points of the tested feature, and finding their correlation factors with the desired geometry (ANSI, 1993, Carr, 1995).

Order dependent approaches use parametric surfaces, attracted by a subset of the sampled points (the control points): Beziers, B-splines, NURBS, etc (Farin, 1990, Mortenson, 1985). One of the most common errors in automated surface fitting is the trend to smooth all the blends, even in regions in which the real object was not intended to be smooth. The issue of intelligent automated enforcement of continuity is still an open problem in Computer Aided Geometric Design (CAGD).

The algorithmic approach places the emphasis on classification and partition of the point set. It is most needed in industrial applications, since it is preparatory for mathematical methods. For example, fitting of Bezier or B-spline curves renders completely different results as different *orderings* in the control points are used.

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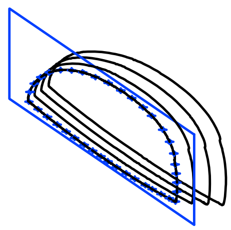

Figure 1: Point classification in coplanar subsets.

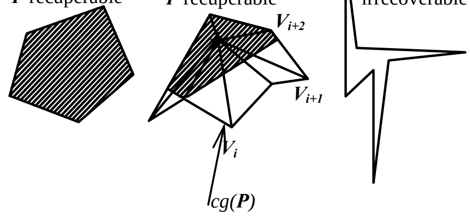

Figure 2 . Recovery of polygon ***P*** according to convexity type.

Alpha shapes (Edelsbrunner, 1994) help to reconstruct surface and volume for a given point cloud, according to the selected alpha parameter. Since they present only C^0 continuity among them, some investigators (Guo, 1997) use alpha shapes as an intermediate step for smooth surfacing. In contrast, this investigation will focus on direct, selectively (C^1 and C^2) continuous surfacing, supported by geometric reasoning tools (Preparata, 1985, Posada, 1997, Ruiz, 1995) to preprocess the point data set to fit as few as possible parametric surfaces to the data. Therefore, it can be seen as an hybrid application of the mathematical and algorithmic approaches. The geometric reasoning tasks are implemented on top of modeling engines as reported in (Ruiz, 1994, Ruiz, 1995).

3. Geometric Reasoning Tools

Along the following discussion it must be stressed that neighboring points *on the object* may appear as scattered samples *in the point set*. Therefore, algorithms to recover the neighborhood information have to be implemented. In the following discussion notation **{ }** represents an unstructured set of elements, **[ ]** corresponds to an ordered list and **{{ }}** denotes a set of sets of objects

3.1 POINT - SURFACE CLASSIFICATION

Since many digitizations are performed in pre-defined trajectories (for example by locking one degree of freedom of the digitizer), the points lie on parallel planes (Fig.1). The function *planar classification* takes an unstructured set ***S*** of points in $\boldsymbol{E}^3$ (***{point}***) and a family of ***N*** digitizing planes, defined by the vector ($\hat{\boldsymbol{u}}$) normal to all planes. It returns a partition ***P*** of the initial set, formed by subsets Π_i of ***S***:

$$\Pi=\{\Pi_1,\Pi_2,\Pi_3,...,\Pi_N\}\text{ , with }\Phi=\Pi_j\cap\Pi_k\text{ for all }j\neq k\text{ , and }S=\bigcup_{j=1}^{j=N}\Pi_j \qquad (1)$$

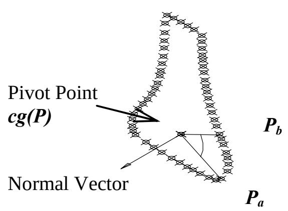

Figure 3. Recovery of a cross section ***P*** of the last.

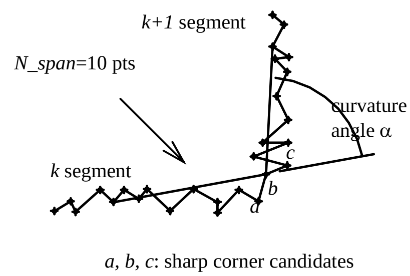

Figure 4. Parameters for corner detection algorithm.

The type of the partition is ***{{point}}***. Each set Π_i in the partition is formed by the points contained in the i^{th} plane of digitization, each specific plane being fixed by its pivot point. The common normal of the digitization planes is (Preparata, 1985):

$$\hat{u} = (n_x, n_y, n_z) / \| (n_x, n_y, n_z) \| \tag{2}$$

$$n_x = \sum_{i=0}^{i=N} (y_i - y_{i+1})(z_i + z_{i+1}) \quad ;$$

$$n_y = \sum_{i=0}^{i=N} (z_i - z_{i+1})(x_i + x_{i+1}) \quad ;$$

$$n_z = \sum_{i=0}^{i=N} (x_i - x_{i+1})(y_i + y_{i+1})$$

Points are considered coplanar if they are within an ε normal distance from the plane. Allowing a coplanarity deviation is a common practice in tolerancing, where planarity of a surface is a statistical rather than deterministic quality (ANSI, 1993, Carr, 1995).

3.2 POLYGON RECOVERY. ANGULAR SORTING ABOUT THE KERNEL

From a set of (coplanar) points ***Q*** randomly sampled on a cross section of an object, it is required to recover the ordered list ***P*** (***[point]***) which represents the cross section. It is immediately clear that, as stated, this problem has no solution. However, if the cross section ***P*** of the object has the properties mentioned below (Preparata, 1985, Posada, 1997), the problem becomes tractable.

Definition: Given a planar, ordered set of points representing a polygon $P=\{p_0,p_1,p_2,..p_n\}$, its kernel, ***Ker(P),*** is the locus of the points ***a*** inside ***P*** such that for all points ***b*** in the boundary of ***P***, the segment ***ab*** lies inside ***P*** (all points ***a*** from which the boundary of ***P*** is visible).

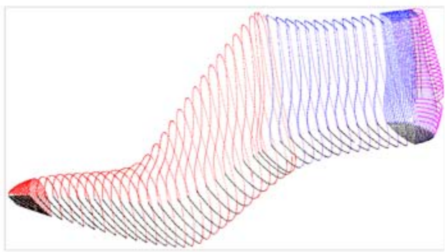

Figure 5. Initial data set from digitization.

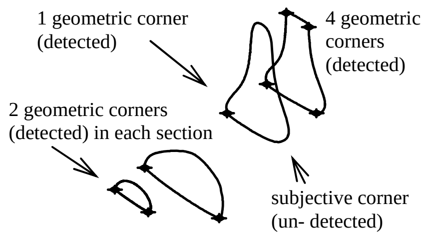

Figure 6. Results of corner detection algorithm.

Corollaries: For all ***P***, ***Ker(P)*** is convex and ***Ker(P)*** $\subseteq$ ***P***. If ***P*** is convex, then ***Ker(P) = P***. If ***Ker(P)*** is not empty, any point ***q*** inside ***Ker(P)*** can be used (along with ***û***, the normal to the plane) as polar origin to angularly sort ***Q***. The sequence so obtained is ***P***, with a possibly different starting point.

A heuristic approach can be used to recover a polygon ***P*** from a random enumeration of its vertices ***Q*** if two hypotheses are assumed: (i) the cross section of the sampled object has non empty kernel, (ii) the center of gravity of ***P***, ***cg(Q)=cg(P)*** lies inside ***Ker(P)***. As shown in Fig.2 and Fig.3, in such cases the angular sort about ***cg(Q)*** enables the recovery of ***P***. The hypotheses just mentioned hold in many engineering applications: objects frequently have sections ***P*** which are either convex or have non empty kernel ***Ker(P)***. If ***cg(P)*** is not inside ***Ker(P)***, the designer may be prompted for a point within it. The treatment of the cases in which the cross section has empty kernel are not considered in this work.

3.3 CORNER DETECTION

Detection of sharp corners of polygon ***P*** is required to avoid enforcement of non-existent C^1 or C^2 continuity conditions on such corners. Since a digitization is a discrete process, every point of ***P*** is, in strict sense, a sharp corner because in each one C^1 continuity is lost. Corner detection algorithm takes a point list (***[point]***) of digitized data, and two parameters, displayed in Fig.4: (a) an angle α, the threshold of tangent discontinuity, above which a sharp edge is recognized, and (b) a re-sampling interval or span ***N_span***, which isolates the estimation of tangent discontinuity from accidental irregularities of the digitization process. Notice that the detected corner could have been ***a***, ***b*** or ***c*** since there is no additional argument to chose one of the three points.

3.4 RE-SAMPLING

Resampling can be used to generate an “artificial” digitization with different sampling parameters than the original one. Different cases are: (a) **Re-sampling by distance**: Given a polygon ***P*** in E^3, and a distance ***d,*** it returns a polygon ***P’*** resulting from

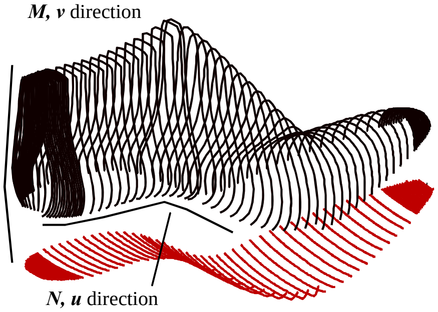

Figure 7. Control polyhedra with N_u x M_v points.

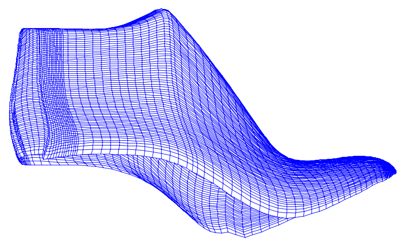

Figure 8. B-spline surface fit for upper part of the last.

resampling the boundary of $\boldsymbol{P}$ in segments of length $\boldsymbol{d}$. (b) **Re-sampling by number of points**: Given a polygon $\boldsymbol{P}$ in $\boldsymbol{E}^3$, and an integer $\boldsymbol{N}$, it returns a polygon $\boldsymbol{P'}$, formed by the vertices of $\boldsymbol{P}$ sampled every $\boldsymbol{N}$ points. (c) **Re-sampling by entity intersection**: Given a polygon $\boldsymbol{P}$ in $\boldsymbol{E}^3$, and a list of entities $\boldsymbol{L}=[e_0,e_1,\ldots e_m]$, this procedure returns a polygon $\boldsymbol{P'}$, formed by the intersections of $\boldsymbol{P}$ against entities e_0, e_1,...etc. Currently, this operation is implemented with $\boldsymbol{L}$ being a family of planes with common normal and different pivot points.

3.5 FILTERING

Filtering is the process by which a function $\boldsymbol{f(\ )}$ is applied to a subsequence $\boldsymbol{S}=[p_i,p_{i+1},p_{i+2},\ldots p_{i+W-1}]$ of $\boldsymbol{W}$ vertices of a polygon $\boldsymbol{P}$, returning vertex q_i, to form the sequence $\boldsymbol{S'}=[q_m,q_{m+1},q_{m+2},\ldots q_h]$. The points of $\boldsymbol{S'}$ are not necessarily in $\boldsymbol{S}$, and $\boldsymbol{S'}$ may be smaller in size than $\boldsymbol{S}$ depending on how the window $\boldsymbol{W}$ is managed. The filter mask used calculates a new point as:

$$\vec{q}_i = f(\vec{p}_i \cdots \vec{p}_{i+W-1}) = \sum_{j=1}^{j=W} \alpha_j \cdot \vec{p}_{j+i-1} \qquad (3)$$

$$\text{where} \quad \sum_{j=1}^{j=W} \alpha_j = 1.0 \;, \alpha_j \geq 0 \;,\; j=1..W$$

with $\alpha_1 = \alpha_2 = \ldots \alpha_N = 1/W$ for averaging, and $\alpha_1 = 1,\ \alpha_2 = \alpha_3 = \ldots.\alpha_W = 0$ for resampling.

4. Case Study

The original data set from a last digitization (Fig.5) contains 33000 points. Notice that: (i) the last was sampled with three different digitization patterns, and (ii) curvature discontinuity between the sole and the upper part requires to fit separate surfaces to them. Therefore, in this case automated surfacing would have immediate limitations. In contrast, with the tools proposed the designer is able to identify, separate and process several point sets, to respond to the above considerations, as follows:

Point - Surface Classification. An initial partition of the point set classifies the points into subsets of coplanar points. Based on the point set, the normal vector $\hat{\boldsymbol{u}}$ is determined, which is common to all the digitizing planes. Fig.1 shows the plane of digitization found by formula (1) (the trajectories shown are very tightly sampled point sets).

Polygon Recovery. Figs. 2 and 3 display polygon recovery. The (non convex) polygon shown: (i) has non-empty kernel, and (ii) its center of gravity is inside the kernel of the polygon. Under those conditions the algorithm for angular sort can be applied. The normal vector $\hat{\boldsymbol{u}}$ is used to enforce an order on the points, according to the right hand rule.

Corner Detection. Fig.6 shows the recognition of sharp edges for different regions of the last. It is found that corner detection works very efficiently for the front and back part of the last. In the central part (foot arc) the algorithm is unable to identify the inner sharp edges of the sole, since they do not exist in the geometric sense. Corner detection was used for removal of the sole (see Fig.7).

Re-sampling. In the present example, re-sampling by number of points was applied with two purposes: (i) to diminish the size of the data set, and (ii) to provide a $N_u \times M_v$ rectangular array of (x,y,z) points as the control polyhedra for a B-spline surface covering the upper part of the last. Each polygon was then resampled to 50 points (Fig.7).

Surface Fitting. The previous tools built 51 planar polylines with 50 points each. A parametric B-spline surface was fit with $N_u \times M_v$ = 51x50 control points, with order 3 in u (N_u) direction , order 4 in v (M_v) direction and standard knot vectors in both directions (Mortenson, 1985). Fig.8 shows a rendering of the B-spline surface obtained. Notice that the heel region required the generation of an artificial digitization resampled by entity intersection from the original one.

5. Conclusions

This article has presented a set of generic tools which take advantage of geometric properties of digitized data in helping a designer in surface fitting tasks. Geometric reasoning allows the separation and sorting of parts of the point data set to conform to functional and geometrical criteria. The software described has allowed to start from raw digitization data to arrive to the surface without using the proprietary tools of the hardware - software maker. Comparing the initial data set of 33.000 points with the 2.550 control points of the surface being fit, significant data reductions can be assessed. The fact that the tools have been written as AIS compatible client software makes them portable across other geometric modeling servers.

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